# What is the Birch and Swinnerton-Dyer conjecture? BMS at Urania (after BMS friday talk) 

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January 23, 2009


Elliptic curves - which can be given by equations of degree 3 (e.g. $X^{3}+Y^{3}=1$ ) - are the most interesting among all algebraic curves. It is an old question in number theory, called a Diophantine problem, to determine the set of rational points on such a curve. The elliptic case, again, is the most interesting and mysterious of all Diophantine problems of dimension 1. For example there may or may not be infinitely many rational solutions. Up to now no algorithm is known for determining the case. However, already in the 60's, Birch and Swinnerton-Dyer experimentally found a deep and mysterious relation of this question to analytic properties of the zeta-function of the curve, which encodes the easily determined solutions of the congruences (e.g. $X^{3}+Y^{3} \equiv 1$ modulo $N$ ). It later became one of the most famous conjectures of mathematics, and is one of the millenium prize problems, for whose solution the Clay Mathematical Institute offers a reward of $\$ 1,000,000$.



