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Discrete Mathematics
First Review WS 07/08

Name: Matrikelnummer:

Before starting, write your name and registration number.

The use of any reference material is strictly forbidden.

This review consists of 4 pages including this page. Please check that you have them all!

Exercise	Score	Max Score
1.a		2
1.b		2
1.c		2
1.d		2
2.a		3
2.b		3
3		6
Total		20

Exercise 1 **2+2+2+2 Punkte**

Give small examples of each of the following types of linear programming problems. Justify your answer.

a) Infeasible,
Answer:

$$\begin{aligned}
&\max\ x \\
&\text{s.t.} \\
&\ x \leq -1, \\
&\ x \geq 0, \\
&\ x \in \mathcal{R}
\end{aligned}$$

b) Unbounded,
Answer:

$$\begin{aligned}
&\max\ x \\
&\text{s.t.} \\
&\ x \geq 0, \\
&\ x \in \mathcal{R}
\end{aligned}$$

c) With a unique optimal solution,
Answer:

$$\begin{aligned}
&\min\ x \\
&\text{s.t.} \\
&\ x \geq 0, \\
&\ x \in \mathcal{R}
\end{aligned}$$

d) With infinitely many optimal solutions,
Answer:

$$\begin{aligned}
&\max\ x + y \\
&\text{s.t.} \\
&\ x + y \leq 1, \\
&\ (x, y) \in \mathcal{R}^2
\end{aligned}$$

Exercise 2 **3+3 Punkte**

Consider the following linear programming problem (LP)

$$\begin{aligned} &\max\ c^T x \\ &\text{s.t.} \\ &\quad Ax \leq b \\ &\quad\ \ x \geq 0 \end{aligned}$$

a) Give the dual of (LP).
Answer:

$$\begin{aligned} &\min\ b^T y \\ &\text{s.t.} \\ &\quad A^T y \geq c \\ &\quad\quad\ y \geq 0 \end{aligned}$$

b) Give a small linear problem for which the primal and its dual are identical.
Answer:
The dual is also given by

$$\begin{aligned} &\max\ (-b^T)y \\ &\text{s.t.} \\ &\quad (-A^T)y \leq -c \\ &\quad\quad\quad\quad\ y \geq 0 \end{aligned}$$

If the primal and its dual are identical, then $b = -c$ and $A^T = -A$
Small example:

$$c = (1,1)^T,\ A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \text{ and } b = (-1,-1)^T.$$

Exercise 3 **6 Punkte**

Given a vertex v of a polyhedron $P = \{x \in R^n : Ax \leq b, x \geq 0\}$, prove that there exists a cost vector $c \in R^n$ such that the unique optimal solution of the LP $\max\{c^T x : x \in P\}$ is attained at v.

Answer:

Let v be a vertex of a polyhedron $P = \{x \in R^n : Ax \leq b, x \geq 0\}$. v is then a face of P with dimension equal to 0. Hence, there exists a vector $\alpha \in \mathcal{R}^n$ and a scalar $beta \in \mathcal{R}$ such that the inequality $\alpha^T x \leq \beta$ is valid for P ($P \subseteq \{x \in \mathcal{R}^n \mid \alpha^T x \leq \beta\}$) and $\{v\} = P \cap \{x \in \mathcal{R}^n \mid \alpha^T x = \beta\}$. Let $c = \alpha$ and consider the following LP

$$\max\{c^T x : x \in P\} \ (I)$$

Since $c^T x \leq \beta$ for every $x \in P$, $c^T v = \beta$ and $v \in P$, v is an optimal solution for the LP (I). Moreover, the set of optimal solutions for the LP (I) is the face $F = P \cap \{x \in \mathcal{R}^n \mid c^T x = \beta\}$, which is equal to the vertex v. Then v the unique optimal solution of the LP (I).