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Discrete Mathematics in Bioinformatics (P1)
Final exam WS 07/08

Name: Matrikelnummer:

You can write your answers in German or English.

A1	A2	A3	A4	$\sum$
15	13	10	7	45

Exercise 1: Linear Programming — 15 points

Solve the following linear program with the Simplex method. Draw the feasible region and show all steps graphically.

$$\begin{aligned} \max \quad & 2x_1 \\ \text{s.t.} \quad & x_2 - x_1 \geq -1 \\ & 2x_1 + x_2 \leq 4 \\ & x_2 \leq 2 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Exercise 2: Combinatorial Optimization — 8 + 5 = 13 points

Given a graph $G = (V, E)$, two vertices $s, t \in V$, and a set of pairs of vertices $C \subset V \times V$, the *shortest antisymmetric path problem* consists in finding a path from s to t with the minimal number of edges, which contains at most one vertex from each pair of vertices in C.

(a) Give an integer linear programming formulation for the antisymmetric shortest path problem.

(b) Suppose you want to apply Lagrangian relaxation. What does the relaxed problem look like for your formulation?

Exercise 3: Network Flow — 3 + 7 = 10 points

Let $G = (V, E)$ be a bipartite graph and let M be a matching in G.

(a) Give definitions for *bipartite*, *matching*, and *maximum matching*.

(b) Describe a method to check in time $O(|V| + |E|)$ whether M is maximum. Explain why your method works and analyze its running time.

Exercise 4: Hashing — 3 + 4 = 7 points

(a) Give pseudocode for the procedure `search`(T, k), which searches an element with key k in a hash table T using *open addressing*.

(b) How long does it take *in the worst case* to insert n elements into an empty hash table using open addressing with linear probing. Justify your answer.