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Discrete Mathematics WS 07/08

Homework 9 (due 18/01)

Exercise 1:
Consider the IP

$$\begin{array}{lll} \text{minimize} & c^T x & \\ \text{subject to} & Ax \geq b & \text{complicating constraints} \\ & Dx \geq d & \text{nice constraints} \\ & x \text{ integral} & \end{array}$$

and let $X = \{x \text{ integral} \mid Dx \geq d\}$. For a fixed $\lambda \geq 0$, we define the problem LR(λ) by

$$\begin{array}{ll} \text{minimize} & c^T + \lambda(b - Ax) \\ \text{subject to} & x \in X \end{array}$$

LR(λ) is called the *Lagrangian relaxation* of IP. In the following, we show that solving LR(λ) may sometimes lead to an optimal solution of IP.

Given a $\lambda \geq 0$, prove that if

a) $x(\lambda)$ is an optimal solution of LR(λ),

b) $Ax(\lambda) \geq b$,

c) $(Ax(\lambda))_i = b_i$ whenever $\lambda_i \geq 0$,

then $x(\lambda)$ is an optimal solution of IP.

Exercise 2:
Given a directed graph $G = (N, E)$ in which every arc $(i, j) \in E$ is associated with two nonnegative integers: its length l_{ij} and its traversal time t_{ij}, the *Constrained Shortest Path Problem (CSPP)* is to find a path from the source s to the sink t such that the total length is minimal and the total transit time is less than some bound T.

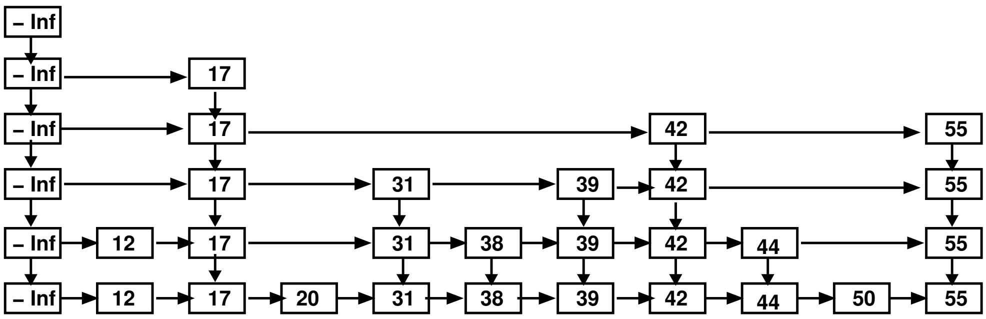
a) Model CSPP as an integer program.

b) Formulate a lagrangian relaxation by dualizing the constraint on the total traversal time.

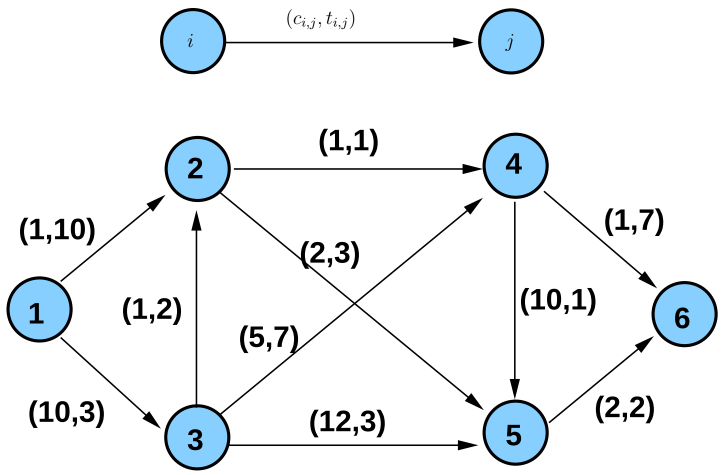
c) Given the graph below, determine graphically a lower bound of the length of the shortest path from node 1 to node 6 with a transit time at most 10 ($T = 10$).

Exercise 3:
Given the skip list below, do the following operations:

- Delete(38),
- Insert(48) (with 2 coin flips),
- Insert(24) (with 3 coin flips),
- Delete(55).



Exercise 4:
Suppose we are using a skip list in a program to store n records. These records will be supplied to the program in random order. As each element is received, it will be inserted into the skip list. What is the expected complexity for putting all n elements into the list?



$(c_{i,j}, t_{i,j})$
i
j
1
2
3
4
5
6
(1,10)
(10,3)
(1,2)
(1,1)
(2,3)
(5,7)
(12,3)
(10,1)
(1,7)
(2,2)