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Discrete Mathematics WS 07/08
Homework 7 (due 14/12)

Exercise 1:
Prove the following lemmas from the lecture.

Lemma.
Let $G = (V, E, H, I)$ be a SEAG with n alignment edges and m interaction matches. Then

- $P_{\mathcal{R}}(G)$ is full-dimensional and
- the inequality $x_i \leq 1$ is facet-defining iff there is no $e_j \in E$ in conflict with e_i.

Lemma.
Let $G = (V, E, H, I)$ be a SEAG with n alignment edges and m interaction matches.

1. The inequality $x_i \geq 0$ is facet-defining iff e_i is not contained in an interaction match.
2. For each interaction match $m_{i,j}$ the inequality $y_{ij} \geq 0$ is facet-defining.

Exercise 2:
Prove that the number of clique inequalities arising from a complete bipartite graph $K_{p,q}$ is $\binom{p+q-2}{p-1}$.

Hint: Develop a recurrence relation for the number of source-sink paths in $PG(K_{p,q})$ and use complete induction to prove the statement.

Exercise 3:
Consider the problem of tiling a planar region R with n dominoes. Each domino is a 2×1 rectangle. R is an arbitrary collection of $2n$ 1×1 squares. Figure 1 shows one example of such a region. The squares are numbered 1 through $2n$. R is described by the set of all the pairs (a, b), $a, b \in \{1, 2, \ldots, 2n\}$, $a < b$, such that square a and square b are edge-connected (i.e., have an edge in common). In this representation, let $R = \{p_1, p_2, \ldots, p_r\}$, where each p_k , for $k = 1$ to r, is a pair of edge-connected squares.

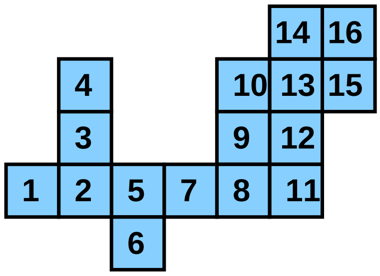


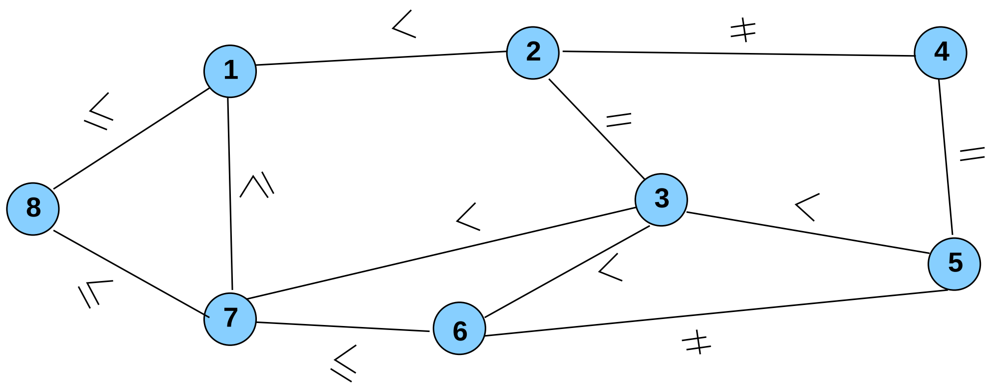
Abbildung 1: Example of a region R where $2n = 16$. A description of R is the set $\{(1,2),(2,3),(3,4),(2,5),\ldots,(14,16),(15,16)\}$

a) Model the problem as a constraint satisfaction problem where the dominoes are the variables, that is, define the variable domains and the constraints.

b) Model the problem as a constraint satisfaction problem where the squares are the variables.

Exercise 4:

Consider the following network



Assume that each variable (node) has a domain of $\{1, 2, 3, 4\}$.

a) Model the problem as a constraint satisfaction problem

b) Apply arc consistency to reduce the domains of the variables.

c) What further reduction can be obtained by fixing the value of the node 5 to the minimum possible value?