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Discrete Mathematics WS 07/08

Homework 4 (due 16/11)

Exercise 1:
When modeling two events e_1 and e_2 with binary variables x and y, respectively, we often use the following requirements:

a) At most one event occurs.

b) Neither or both events occur.

c) If e_1 occurs then e_2 occurs.

d) e_1 occurs only if e_2 occurs.

Define a linear constraint for each requirement.

Exercise 2:
In the classical *assignment problem*, we are given n people, m tasks and an $m \times n$-matrix C such that each coefficient C_{ij} is equal to the cost of assigning person j to task i. Each task is assigned to one person, and each person can do at most one task. The objective is to minimise the total cost of the assignments. Model this problem using combinatorial optimization.

Exercise 3:
Solve graphically the following ILP:

$$
\begin{aligned}
&\max\ 2x + 3y \\
&\text{s.t. to} \\
&\begin{array}{rcrcl}
-x & + & y & \leq & 5, \\
x & + & 3y & \leq & 35, \\
 & & x & \leq & 20, \\
 & & x, y & \geq & 0, \\
 & & x, y & \multicolumn{2}{l}{\textit{integer}.}
\end{array}
\end{aligned}
$$

Exercise 4:
This exercise is obligatory! Please print or write down your answers.
Consider the following problems:

- **Shortest path:** Given a weighted directed graph and two distinct vertices s and t, find the length of the shortest path from s to t.
- **Optimal laptop distribution:** Given one long row of students S and a subset $L \subseteq S$ with students who have a laptop, find a distribution of the laptops such that:
  - Each student either has a laptop or is sitting next to somebody who has a laptop.
  - The number of laptop owners that have to move is minimal

a) Formulate an integer linear program for each problem.

b) Model this two problems with ZIMPL and solve them with SCIP. The necassary data and tools can be found on the homepage.