Dr. Gunnar W. Klau
Abdelhalim Larhlimi
Institut für Mathematik II
AG Mathematik in den Lebenswissenschaften

Discrete Mathematics WS 07/08

Homework 3 (due 09/11)

Exercise 1:
Prove that exactly one of the following two cases holds for the LP $max\{c^T x : Ax \leq 0, x \geq 0\}$:

a) $x^* = 0$ is an optimal solution.

b) LP is unbounded.

Exercise 2:
Consider the following primal problem

$$
\begin{array}{rcrcrcl}
\multicolumn{7}{l}{\max x_1 + 4x_2 + 5x_3} \\
\multicolumn{7}{l}{\text{s.t.}} \\
-x_1 & + & x_2 & + & x_3 & \leq & 4 \\
3x_1 & + & x_2 & + & x_3 & \leq & 16 \\
 & & x_2 & & & \geq & 1 \\
x_1, & & x_2, & & x_3 & \geq & 0
\end{array}
$$

Without using the Simplex Method, find optimal primal and dual solutions from the following list. Explain your reasoning.

$$
\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}
\begin{bmatrix} 3 \\ 0 \\ 7 \end{bmatrix}
\begin{bmatrix} \frac{7}{2} \\ \frac{3}{2} \\ 15 \end{bmatrix}
\begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix}
\begin{bmatrix} 3 \\ -1 \\ 8 \end{bmatrix}
\begin{bmatrix} \frac{7}{2} \\ \frac{3}{2} \\ 1 \end{bmatrix}
\begin{bmatrix} \frac{7}{2} \\ \frac{3}{2} \\ 0 \end{bmatrix}
\begin{bmatrix} -1 \\ 12 \\ 7 \end{bmatrix}
$$

Exercise 3:

a) Consider an LP in canonical form, i.e.

$$\textbf{(LP)} \qquad \begin{array}{rrcl} max & c^T x & & \\ s.t. & Ax & \leq & b \\ & x & \geq & 0 \end{array}$$

and the corresponding dual problem

$$\textbf{(DP)} \qquad \begin{array}{rrcl} min & b^T y & & \\ s.t. & A^T y & \geq & c \\ & y & \geq & 0 \end{array}$$

Show that the dual of the dual problem (DP) corresponds to the primal problem (LP).

b) prove that it is impossible to have primal and dual both unbounded

Exercise 4:
Consider the polyhedron P described by

$$\begin{array}{rcrcr} x_1 & - & x_2 & \leq & 0 \\ -x_1 & + & x_2 & \leq & 1 \\ & & 2x_2 & \geq & 5 \\ 8x_1 & - & x_2 & \leq & 16 \\ x_1 & + & x_2 & \geq & 2 \end{array}$$

a) Find the dimension of P

b) Describe all the faces of P

c) Give a 'minimal' representation of P (a representation that uses just the facet-defining inequalities)

Exercise 5:

a) Why are we so concerned with vertices?

b) Is the following statement true? The set of optimal solutions to a linear program must form a face of the feasible polyhedron. Prove your answer.

c) Is it possible to start with a full-dimensional polyhedron (dimension n), enforce one of its inequalities as an equality and have a nonempty polyhedron with dimension two lower than the original polyhedron (dimension $n - 2$)? If so, give an example. If not, give an argument why not.