## Discrete Mathematics WS 07/08 <br> Homework 2 (due 02/11)

## Exercise 1:

Consider the following LP

$$
\begin{array}{lll}
\max & -x_{1}-x_{2} & \\
\text { s.t. } & -2 x_{1}-x_{2} & \leq 4, \\
& -x_{1}+3 x_{2} & \leq-7, \\
& x_{1}, x_{2} & \geq 0 .
\end{array}
$$

Transform it into $\max \left\{\bar{c}^{T} \bar{x}: \bar{A} \bar{x}=b, \bar{x} \geq 0\right\}$, and answer these questions: How many partitions $(\mathcal{B}, \mathcal{N})$ of the variables with $|\mathcal{B}|=m,|\mathcal{N}|=n$ are there? What are they? Which of them correspond to basic feasible solutions?

## Exercise 2:

a) Consider a linear programming problem $\min \left\{c^{T} x: A x \leq b\right\}$. Given a feasible solution $x$, a vector $d$ is a feasible direction at $x$ if there exists some $\theta>0$ such that $x+\theta d$ is a feasible solution. Prove the following:
i) A feasible solution $x$ is optimal if and only if $c^{T} d \geq 0$ for every feasible direction $d$ at $x$.
ii) A feasible solution $x$ is the unique optimal solution if and only if $c^{T} d>0$ for every nonzero feasible direction $d$ at $x$.
iii) Let $P=\left\{x \in \mathbb{R}^{3} \mid x_{1}+x_{2}+x_{3}=1, x \geq 0\right\}$ and consider the vector $x=(0,0,1)$. Find the set of feasible directions at $x$.

## Exercise 3:

Solve the following linear programming problem using the simplex method. Draw the feasible region and show all the steps graphically.

$$
\begin{aligned}
& \max 2 x+3 y \\
& \text { s.t. to } \\
& \begin{aligned}
-x+y & \leq 5, \\
x+3 y & \leq 35, \\
x+ & \leq 20, \\
x, y & \geq 0
\end{aligned}
\end{aligned}
$$

## Exercise 4:

Using the first phase of the simplex metho, find a feasible solution of

$$
\begin{aligned}
& \max 3 x+y \\
& \text { s.t. to } \\
& x-y \leq-1, \\
& -x-y \leq-3, \\
& 2 x-y \leq 2, \\
& x, y \geq 0 \text {. }
\end{aligned}
$$

