Dr. Gunnar W. Klau Abdelhalim Larhlimi Institut für Mathematik II AG Mathematik in den Lebenswissenschaften



Discrete Mathematics WS 07/08 Homework 2 (due 02/11)

Exercise 1: Consider the following LP

$$\begin{array}{rcl}
\max & -x_1 - x_2 \\
s.t. & -2x_1 - x_2 & \leq 4, \\
& -x_1 + 3x_2 & \leq -7, \\
& x_1, x_2 & \geq 0.
\end{array}$$

Transform it into $max\{\overline{c}^T\overline{x}: \overline{A}\overline{x} = b, \overline{x} \ge 0\}$, and answer these questions: How many partitions $(\mathcal{B}, \mathcal{N})$ of the variables with $|\mathcal{B}| = m$, $|\mathcal{N}| = n$ are there? What are they? Which of them correspond to basic feasible solutions?

Exercise 2:

- a) Consider a linear programming problem $min\{c^Tx : Ax \leq b\}$. Given a feasible solution x, a vector d is a *feasible direction* at x if there exists some $\theta > 0$ such that $x + \theta d$ is a feasible solution. Prove the following:
 - i) A feasible solution x is optimal if and only if $c^T d \ge 0$ for every feasible direction d at x.
 - ii) A feasible solution x is the unique optimal solution if and only if $c^T d > 0$ for every nonzero feasible direction d at x.
 - iii) Let $P = \{x \in \mathbb{R}^3 \mid x_1 + x_2 + x_3 = 1, x \ge 0\}$ and consider the vector x = (0, 0, 1). Find the set of feasible directions at x.

Exercise 3:

Solve the following linear programming problem using the simplex method. Draw the feasible region and show all the steps graphically.

$$\max 2x + 3y$$

s.t. to
$$-x + y \leq 5,$$

$$x + 3y \leq 35,$$

$$x + \leq 20,$$

$$x, y \geq 0.$$

Exercise 4:

Using the first phase of the simplex metho, find a feasible solution of

$$\max 3x + y$$

s.t. to
$$x - y \leq -1,$$

$$-x - y \leq -3,$$

$$2x - y \leq 2,$$

$$x, y \geq 0.$$