Dr. Gunnar W. Klau
Abdelhalim Larhlimi
Institut für Mathematik II
AG Mathematik in den Lebenswissenschaften

Freie Universität Berlin

Discrete Mathematics WS 07/08

Homework 1 (due 26/10)

Exercise 1:

A set $C \subset \mathbb{R}^n$ is *convex* if for any two points $x_1, x_2 \in C$ and for any $\lambda \in [0,1]$, $\lambda x_1 + (1-\lambda)x_2 \in C$.

a) Prove that the set $\{x \in \mathbb{R}^n \mid Ax \leq b\}$, if nonempty, is convex for any matrix $A \in \mathbb{R}^{m \times n}$ and any vector $b \in \mathbb{R}^m$.

b) Suppose that a linear program has two optimal solutions x_1 and x_2. Prove that if a vector x can be expressed as a convex combination of x_1 and x_2, then x is an optimal solution.

c) Prove that it is impossible for a linear program to have **exactly** two distinct optimal solutions.

Exercise 2:

Let S consist of the following five points in $\mathbb{R}^2$:

$$x^1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix},\ x^2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix},\ x^3 = \begin{pmatrix} 4 \\ -1 \end{pmatrix},\ x^4 = \begin{pmatrix} 2 \\ -2 \end{pmatrix},\ x^5 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}.$$

a) Express x^1 as a convex combination of three of the other points.

b) Give an inequality description of $conv(S)$. That is, find A and b so that $conv(S) = \{x \in \mathbb{R}^2 \mid Ax \leq b\}$.

c) What is the optimal solution for $max\{c^T x : x \in conv(S)\}$ with $c^T = (1, -1)$. Define an other objective function for which there are multiple optimal solutions.

Exercise 3:

Model the following scenario as a linear programming problem. Please, do **not** attempt to solve the problem.

During the 2000 olympics, Marion Jones was accused of using a blend of four banned performance-enhancing drugs : human growth hormone (HGH), anabolic steroids (AS), the endurance booster erythropoietin (EPO), and insulin. Jones had previously been suspended for four years after failing to show up for a random drug test, but her suspension was dropped after an appeal. Suppose that Jones can use a blend of three designer drugs, each of which contains a mixture of the four substances:

	HGH	AS	EPO	insulin
Type A	10	5	20	5
Type B	3	20	10	0
Type C	20	5	5	5

She would like to maximize the total number of units of the four substances she uses, but she doesn't want to get caught or die. To pass the tests for HGH, AS, and EPO, she is required to ingest at most 60, 40, and 100 units of the three drugs, respectively. Besides, it is difficult to test for insulin because the later passes through the body extremely quickly. However, an overdose of insulin results in hypoglycemia, the symptoms of which range from nausea to death. For this reason, Marion should ingest at most 35 units of insulin.

Exercise 4:

Read the MATLAB help of **linprog**, **bintprog** and write the MATLAB codes to solve the two following linear programmings. What are the optimal solutions and optimal objective values?

$$
\begin{array}{lll}
max & z = 2x_1 - x_2 + 4x_3 & \\
s.t. & x_1 + x_2 + x_3 + x_4 & = 2 \\
 & x_2 - x_3 & \geq 0 \\
 & x_3 + x_4 & \leq 1
\end{array}
$$

$$
\begin{array}{lll}
max & z = 2x_1 - x_2 + 4x_3 & \\
s.t. & x_1 + x_2 + x_3 + x_4 & = 2 \\
 & x_2 - x_3 & \geq 0 \\
 & x_3 + x_4 & \leq 1 \\
 & x_1, x_2, x_3, x_4 \in \{0, 1\} &
\end{array}
$$