Minimal violations of the Snoussi-condition.

Ling Sun

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Review

1. .1 Model, $M = (I, K)$, 3 model conditions.
   .2 Equivalent models.
   .3 ASTG, $T = (X, S)$, the dynamics generated by a model M.
   .4 3 u-row types.
   .5 Isomorphic groups of u-rows.
   .6 Extremal states and extremal rows.
2. Algorithms:

The Snoussi-condition in an interaction

Definition

An *interaction* $(u, v) \in E$ of a model $M = (I, K)$ satisfies the *Snoussi-condition*, if

$$\forall \omega \subseteq \mathrm{Pre}(v) \backslash \{u\}, \;\; K(v, \omega) \leq K(v, \omega \cup \{u\}).$$

Definition

The number of violations of Snoussi-condition in all interactions, termed as s:

$$s := \sharp\{(u, v, \omega) \mid (u, v) \in E, \omega \subseteq \mathrm{Pre}(v) \backslash \{u\} \wedge K(v, \omega) > K(v, \omega \cup \{u\})\}.$$

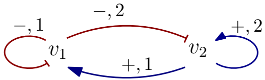
Snoussi1989, Qualitative dynamics of piecewise-linear differential equations: a discrete mapping approach, Dynamics and Stability of Systems.

Definition

(On ω-side of u) (Lorenz2011)

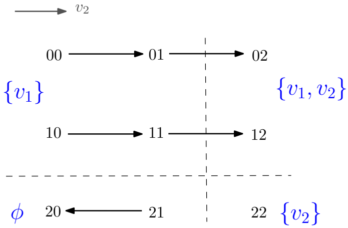
- Given $u \in V$, $\omega \subseteq \mathrm{Pre}(u)$, we say that $a \in \{0, \ldots, max_u\}$ lies *on the ω-side of u* if there exists a state $x \in X$, with $x_u = a$ and $\mathrm{Res}_u(x) = \omega$.

I



K

ω	$K(v_1,\omega)$	ω	$K(v_2,\omega)$
ϕ	0	ϕ	0
$\{v_2\}$	2	$\{v_2\}$	2
$\{v_1\}$	1	$\{v_1\}$	2
$\{v_1, v_2\}$	1	$\{v_1, v_2\}$	2



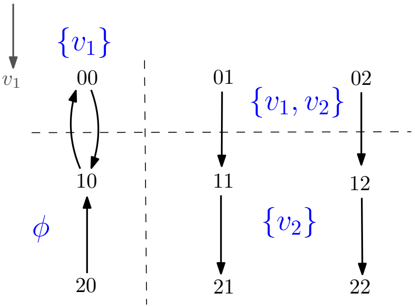
0 and 1 lie on $\{v1\}$-side of v_2

0 and 1 lie on $\emptyset$-side of v_2

2 lies on $\{v1, v2\}$-side of v_2

2 lies on $\{v2\}$-side of v_2

Does $K(v_1, \omega)$ lie on ω-side of v_1?



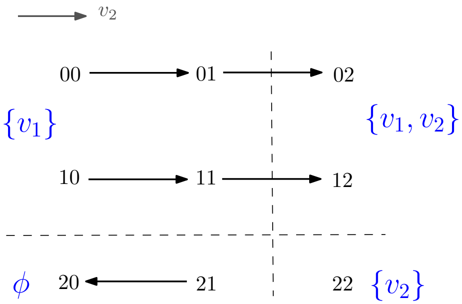
ω	$K(v_1, \omega)$
$\emptyset$	0
$\{v_2\}$	2
$\{v_1\}$	1
$\{v_1, v_2\}$	1

Question: for each $\omega \subseteq Pre(v_1)$, does $K(v_1, \omega)$ lie on the ω-side of v_1?

Answer:

ω	$K(v_1, \omega)$	lies on ω-side?
$\emptyset$	0	no
$\{v_2\}$	2	yes
$\{v_1\}$	1	no
$\{v_1, v_2\}$	1	no

Does $K(v_2, \omega)$ lie on ω-side of v_2?



ω	$K(v_2, \omega)$
$\emptyset$	0
$\{v_2\}$	2
$\{v_1\}$	2
$\{v_1, v_2\}$	2

Question: for each $\omega \subseteq Pre(v_2)$, does $K(v_2, \omega)$ lie on the ω-side of v_2?

Answer:

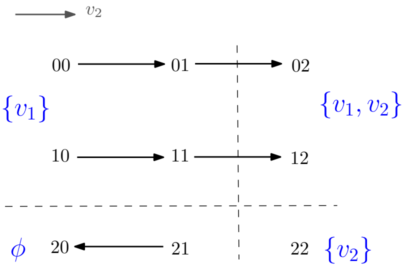
ω	$K(v_2, \omega)$	lies on ω-side?
$\emptyset$	0	yes
$\{v_2\}$	2	yes
$\{v_1\}$	2	no
$\{v_1, v_2\}$	2	yes

How to decide whether $K(u,\omega)$ lie on ω-side of u?

Remark

$K(u,\omega)$ lies on ω-side of u in the following two cases:

- if $u \in \omega$ and $K(u,\omega) \geq \vartheta(u,u)$;
- if $u \notin \omega$ and $K(u,\omega) < \vartheta(u,u)$.



ω	$K(v_2,\omega)$
$\emptyset$	0
$\{v_2\}$	2
$\{v_1\}$	2
$\{v_1, v_2\}$	2

Question: for each $\omega \subseteq Pre(v_2)$, does $K(v_2,\omega)$ lie on the ω-side of v_2?

Answer:

ω	$v_2 \in \omega$	$K(v_2,\omega)$	vs $\vartheta(v_2,v_2) = 2$	lies on ω-side?
$\emptyset$	$\notin$	0	$<$	yes
$\{v_2\}$	$\in$	2	$=$	yes
$\{v_1\}$	$\notin$	2	$=$	no
$\{v_1, v_2\}$	$\in$	2	$=$	yes

To find a logical parameter function satisfying the Snoussi-condition as much as possible, the following lemma [1] is introduced.

Lemma

[1] Given a model $M = (I, K)$, the logical parameter function K^S is defined in the following way:

- *For all $u \in V$, for all $\omega \subseteq \mathrm{Pre}(u)$: if $K(u, \omega)$ lies on the ω side from u*

$$K^S(u, \omega) := K(u, \omega)$$

- *For all $u \in V$, for all $\omega \subseteq \mathrm{Pre}(u)$: if $K(u, \omega)$ does not lie on the ω side from u*

$$K^S(u, \omega) := \begin{cases} \vartheta(u, u) - 1 & \varepsilon(u, u) = +, \ u \in \omega \\ \vartheta(u, u) & \varepsilon(u, u) = +, \ u \notin \omega \\ 0 & \varepsilon(u, u) = -, \ u \notin \omega \\ max_u & \varepsilon(u, u) = -, \ u \in \omega \end{cases}$$

If in $M^S = (I, K^S)$ there is a violation of the Snoussi-condition in component u for some $\omega \subseteq \varsigma \subseteq V$, then the same is true in all its isomorphic models.

Example.

x_u	0	$\dots$	$t-1$	t	$\dots$	max_u
$x \in \tau^u$	•	•	•	•	•	•
$\text{Res}(u,x)$	ω			ς		
$K(u, Res)$	if $K(u, Res)$ does not lie one Res-side of u					
	$K(u,\omega) > t-1$			$K(u,\omega) < t$		
$\varepsilon(u,u) = +$	$u \notin \omega$			$u \in \varsigma$		
$K^S(u, Res)$	$K^S(u,\omega) = t$			$K(u,\omega) = t-1$		
$\varepsilon(u,u) = -$	$u \in \omega$			$u \notin \varsigma$		
$K^S(u, Res)$	$K^S(u,\omega) = max_u$			$K(u,\omega) = 0$		

Figure: Illustration of the Lemma for K^S. t is supposed to be $\vartheta(u,u)$.

References

T. Lorenz, *Vergleich von zwei- und mehrwertigen modellen bioregulatorischer netzwerke*, diplomarbeit, Freie Universität Berlin, 2011.

Discussion about the simulation of:

1. From model to ASTG.
2. Model conditions checking: observability condition and Snoussi-condition.
3. Finding attractors of an ASTG.

Algorithms

1. Algorithm *VisibilityModel*. The following sub-tasks are included.
   - detecting u-row types,
   - Algorithm *LogicalParameter*,
   - Algorithm *ActivityLevel*,
2. Algorithm *Observability-Snoussi-Model*. With similar sub-tasks.