Reverse Engineering Algorithms

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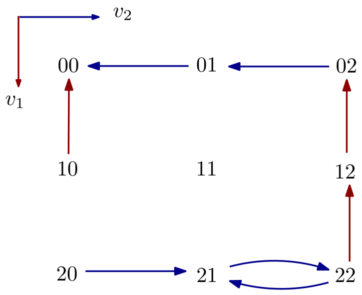
Review

- Model, $M = (I, K)$. (IG I has no multiple edges).
- ASTG, $T = (X, S)$, the dynamics generated by a model M.
- Proposition:3 u-row types.
- Lemma: isomorphic groups of u-rows.

Extremal state and extremal row

Definition

(Lorenz2011) A state $x = (x_u)_{u \in V}$ is called an *extremal state*, if $x_u \in \{0, max_u\}$, for all $u \in V$.
A u-row $\tau^u = (x^0, \ldots, x^{max_u})$ is *extremal*, if both x^0 and x^{max_u} are extremal states.



Lemma

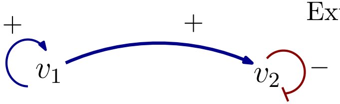
(Lorenz2011)Given a component $u \in V$ and a resource $\omega \subseteq \mathrm{Pre}(u)$, there always exists an extremal state $x \in X$ such that $\mathrm{Res}_u(x) = \omega$.

Proof.

For all $v \in V$, define

$$x_v := \begin{cases} 0 & v \notin \mathrm{Pre}(u) \\ 0 & \varepsilon(v,u) = + \wedge v \notin \omega \\ max_v & \varepsilon(v,u) = + \wedge v \in \omega \\ max_v & \varepsilon(v,u) = - \wedge v \notin \omega \\ 0 & \varepsilon(v,u) = - \wedge v \in \omega \end{cases} \tag{2.1}$$

Then by construction, $\mathrm{Res}_u(x) = \omega$. □



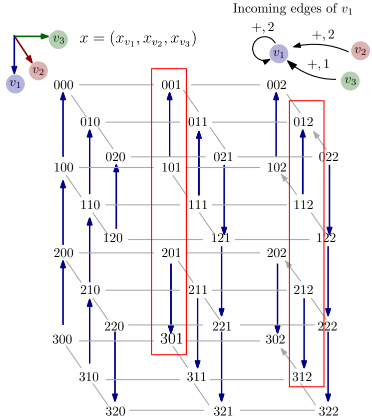
Extremal state of v_1 which has resource $\{v_1\}$

So that, $x = (x_{v_1}, x_{v_2}) = ?$

Isomorphic u-rows with two states.

Lemma

(Lorenz2013) Let $x, y \in X$ such that there exists a component $u \in V$ with $Res_u(x)\backslash u = Res_u(y)\backslash u$. Then the u-row $(x^0, \cdots, x^{max_u})$ containing x is isomorphic to the u-row $(y^0, \cdots, y^{max_u})$ containing y.



$x = 012 \quad Res_{v_1}(012) = \{v_3\}$

$y = 301 \quad Res_{v_1}(301) = \{v_1, v_3\}$

$Res_{v_1}(x)\backslash v_1 = Res_{v_1}(y)\backslash v_1$

Theorem

(Lorenz2013) For any model $M = (I, K)$, the state transition graph T_M is uniquely determined by I and the extremal rows of T_M.

Proof.

(Lorenz2013) For any u-row $(x^0, \cdots, x^{max_u})$, one can always find an extremal state y with $Res_u(x^0) = Res_u(y)$, according to Lemma 2. Quite directly, from Lemma of isomorphic u-rows with two states, the extremal row including y is isomorphic to the u-row $(x^0, \cdots, x^{max_u})$. □

Theorem illustration

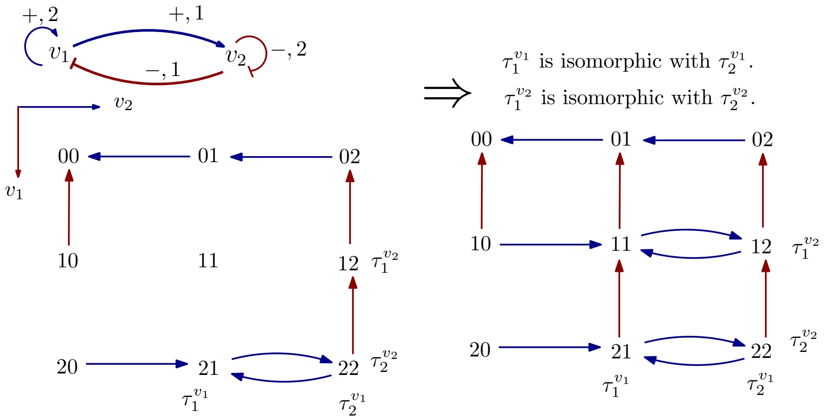
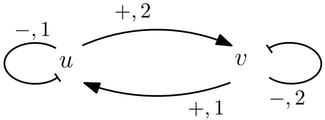


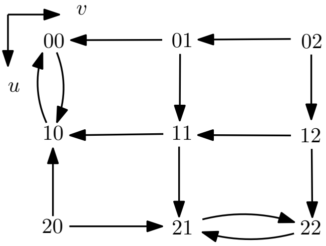
Figure: The interaction graph and the extremal rows of an ASTG uniquely determine the complete ASTG. $\vartheta(v_1, v_2) = 1$ infers that, $\tau_1^{v_2}$ is isomorphic with $\tau_2^{v_2}$. Similarly, $\vartheta(v_2, v_1) = 1$ infers that, $\tau_1^{v_1}$ is isomorphic with $\tau_2^{v_1}$.



(a) I

ω	$K(u,\omega)$	ω	$K(v,\omega)$
ϕ	0	ϕ	0
$\{v\}$	2	$\{v\}$	0
$\{u\}$	1	$\{u\}$	0
$\{u,v\}$	2	$\{u,v\}$	2

(b) K



(c) T

ω	$K'(u,\omega)$	ω	$K'(v,\omega)$
ϕ	0	ϕ	0
$\{v\}$	2	$\{v\}$	0
$\{u\}$	2	$\{u\}$	0
$\{u,v\}$	1	$\{u,v\}$	2

(d) K'

Figure: (a) IG I. (b) Logical parameter function K. (c) Corresponding ASTG T. (d) Alternative logical parameter function K'. $M_1 = (I, K)$ and $M_2 = (I, K')$ are two isomorphic models generating the same ASTG T, where K satisfies the Snoussi-condition but K' does not.

Definition

(Equivalent models) Let $M_1 = (I_1, K_1)$ and $M_2 = (I_2, K_2)$ be two models, where $I_1 = (V, E_1, \varepsilon_1, \vartheta_1, max)$ and $I_2 = (V, E_2, \varepsilon_2, \vartheta_2, max)$. M_1 and M_2 are *equivalent* if

$$\delta_{M^1}(u, x) = \delta_{M^2}(u, x), \forall x \in X,\ \forall u \in V.$$

(Let us see an example for this definition and the following Lemma.)

Lemma

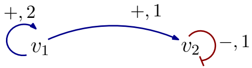
(Lorenz2011) Given a model $M^1 = (I^1, K^1)$ *with* $I^1 = (V, E, \varepsilon^1, \vartheta^1, max)$.

$a)$ *For IG* $I^2 = (V, E, \varepsilon^2, \vartheta, max)$, *a logical parameter function* K^2 *is defined as, for all* $u \in V$ *and* $\omega \in \mathrm{Pre}(u) \mid_{I^1}$

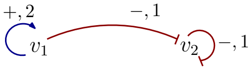
$$K^2(u, \omega) = K^1(u, \omega') \quad \omega' := \omega \Delta \{v \in \mathrm{Pre}(u) \mid \varepsilon^1(v, u) \neq \varepsilon^2(v, u)\}$$

then, the model $M^2 = (I^2, K^2)$ *defines the same ASTG as* M^1.

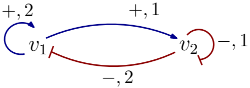
$b)$ *For an IG* $I^3 = (V, E^3 = E \sqcup E', \varepsilon^3, \vartheta^3, max)$, *for all* $u \in V$, *one can define its predecessors in the following way:* $\mathrm{Pre}_E(u) := \{(v, u) \mid (v, u) \in E\}$, $\mathrm{Pre}_{E'}(u) := \{(v, u) \mid (v, u) \in E'\}$, $\mathrm{Pre}_{E^3}(u) := \mathrm{Pre}_E(u) \cup \mathrm{Pre}_{E'}(u)$. *If the thresholds on those interactions from* E, $\vartheta^3 \mid_E$ *is identical with* ϑ, *i.e.,* $\vartheta^3 \mid_E \equiv \vartheta$, *then one can define a* K^3 *as follows:* $\forall u \in V \;\; \forall \omega \subseteq \mathrm{Pre}_{E^3}(u)$*:*
$\omega' := \omega \Delta \{v \in \mathrm{Pre}_E \mid \varepsilon^1(v, u) \neq \varepsilon^3(v, u)\}$, $K^3(u, \omega) := K^1(u, \omega')$
$M^3 := (I^3, K^3)$ *defines the same ASTG as* M^1. *Moreover,* K^3 *is chosen that no edges in* E' *is visible.*



(a) I^1.



(b) I^2.



(c) I^3.

ω	$K^1(v_1,\omega)$
ϕ	0
$\{v_1\}$	1

ω	$K^1(v_2,\omega)$
ϕ	0
$\{v_2\}$	1
$\{v_1\}$	2
$\{v_1,v_2\}$	2

(d) K^1

ω	$K^2(v_1,\omega)$
ϕ	0
$\{v_1\}$	1

ω	$K^2(v_2,\omega)$
ϕ	2
$\{v_2\}$	2
$\{v_1\}$	0
$\{v_1,v_2\}$	1

(e) K^2

ω	$K^3(v_1,\omega)$
ϕ	0
$\{v_2\}$	0
$\{v_1\}$	1
$\{v_1,v_2\}$	1

ω	$K^3(v_2,\omega)$
ϕ	0
$\{v_2\}$	1
$\{v_1\}$	2
$\{v_1,v_2\}$	2

(f) K^3

Figure: (a), (d) $M^1 = (I^1, K^1)$, (b), (e) $M^2 = (I^2, K^2)$, (c), (f)$M^3 = (I^3, K^3)$.

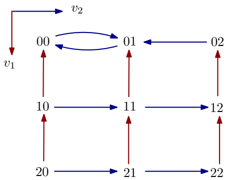


Figure: The ASTG T of the three models in Figure 3.

References

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T. LORENZ, H. SIEBERT, AND A. BOCKMAYR, *Analysis and characterization of asynchronous state transition graphs using extremal states*, Bulletin of Mathematical Biology, 75 (2013), pp. 920–938.