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December 15, 2016

Deadline: January 05, 2016, 11:45 am

Optimization

WS 2016/17

Exercise 1

1. Transform the linear optimization problem

$$\begin{array}{lrcrcl} \min & -x_1 & + & 3x_2 & & \\ \text{w.r.t.} & x_1 & & & \leq & 4 \\ & -2x_1 & + & x_2 & = & 18 \\ & & & x_2 & \geq & 0 \end{array}$$

   to the canonical form $\max\{c^T x \mid Ax = b, x \geq 0\}$.

2. Consider the linear optimization problem:

$$\begin{array}{lrcrcl} \max & x_1 & + & x_2 & & \\ \text{w.r.t.} & x_1 & + & x_2 & \geq & 1 \\ & 3x_1 & - & 2x_2 & \geq & -6 \\ & & & x_2 & \leq & 4 \\ & x_1 & & & \leq & 6 \\ & 2x_1 & + & x_2 & \geq & 0 \\ & & & x_2 & \geq & 0 \end{array}$$

   (a) Determine the feasible region.
   (b) Solve the optimization problem graphically. How many solutions exist and what is the optimal value?
   (c) Solve the problem for the new objective function x_2. How many solutions exist and what is the optimal value?

3. **Income optimization**

   A company produces cat food and dog food. There are three different types of machines which are needed. The first one cut the meat, the second one put some flavour in the food. The third machine packages the food in cans.
   Both types of food need for one unit only 10 minutes on the first machine. Cat food needs 30 minutes on the second machine and packaging takes 50 minutes on the last one.
   Dog food is much less time consuming (dogs are no gourmets): there are 10 minutes needed on the second machine and 30 minutes on the last one.
   One unit of cat food can be sold for 4 Euros, one unit of dog food for 2 Euros.
   Because the factory don't want the cats and dogs to starve they produce at least one unit of each food.
   Since the machines are used for other food too, there is a time limit for each machine:

   - machine 1 can be used for 0.5 hours
   - machine 2 can be used for 1 hours
   - machine 3 can be used for 2.5 hours

   How much cat food and how much dog food should they produce if they want to maximize their income?

   (a) Formulate the problem as a linear program.

   (b) Solve the linear program graphically to compute the coordinates of the optimal solution as well as its value.

Programming Exercises - Deadline January 4th, 2017, 8:00 am

Solve all the given LPs with MATLAB by using *glpk* as a solver. Produce for each exercise an .lp-file to check your programs. For help on glpk check `https://www.gnu.org/software/octave/doc/v4.0.0/Linear-Programming.html`