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Optimization

WS 2015/16

Exercises 5

1. Lagrangean Relaxation I

Consider the following problem

$$\begin{array}{llll} \min & 2x_1 & - & 3x_2 \\ \text{w.r.t.} & 3x_1 & - & 4x_2 \leq -6 \\ & -x_1 & + & x_2 \leq 2 \\ & 6x_1 & + & 2x_2 \geq 3 \\ & 6x_1 & + & x_2 \leq 15 \\ & & & x_1, x_2 \geq 0 \\ & & & x_1, x_2 \in \mathbb{Z} \end{array}$$

- Draw the corresponding polytope and determine graphically the optimal solution Z_{IP} of the original problem and Z_{LP} , the solution of the LP-relaxation.
- Now apply lagrangean relaxation by relaxing the first inequality. Draw the polytope of the relaxed ILP. Determine the set X of feasible solutions for the relaxed problem.
- The new objective function is then:

$$Z(P) = \min_{(x_1, x_2) \in X} 2x_1 - 3x_2 + p(6 + 3x_1 - 4x_2)$$

Calculate $Z_D = \max_{p \geq 0} Z(p)$ and compare this value to Z_{IP} and Z_{LP} . (To obtain Z_D , draw the graphs of the function $f(p) = 2x_1 - 3x_2 + p(-6 - 3x_1 + 4x_2)$ for all $(x_1, x_2) \in X$. Do this by hand.)

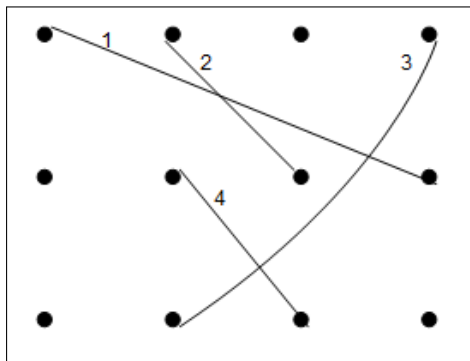
- repeat a-c for the objective functions $-x_1 + x_2$ and $-x_1 - x_2$ and compare Z_{LP} , Z_D , and Z_{IP} . To draw the graphs of the functions $f(p)$ use MATLAB.

2. Lagrangean Relaxation II

Prove Lemma 1 (see script page 4001) stating that (in case of a minimization problem) if $\lambda \geq 0$, then $Z(\lambda) \leq Z_{IP}$, where Z_{IP} is the optimal value of an original ILP and $Z(\lambda)$ is the optimal value of the relaxed problem for a given value of the Lagrangean multiplier λ .

3. Branch and Cut

Given the following alignment graph:



All edges have weight 1.

- (a) Try to solve the alignment problem by using branch-and-cut: Add mixed cycle inequalities (see the ‘shortest path’ method in the script, page 18) to the corresponding (relaxed) LP. Can you reach an optimal solution for the ILP without branching?
- (b) Now use branching to solve the problem.
- (c) Instead of branching, just add the inequality

$$x_1 + x_2 + x_3 + x_4 \leq 2$$

Can you solve the ILP now?

- (d) Prove that the inequality in (c) is facet-defining.