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Deadline: January 21, 2016, 11:45 am

Optimization

WS 2015/16

Exercise 4

1. **Branch and Bound**

$$\begin{array}{lccccccccc} \max & 7x_1 & + & 10x_2 & + & 4x_3 & + & 5x_4 & & \\ \text{w.r.t.} & & & & & & & & & \\ & 6x_1 & + & 8x_2 & + & 4x_3 & + & 2x_4 & \leq & 15 \\ x_1, x_2, x_3, x_4 \in \{0,1\} & & & & & & & & & \end{array}$$

   (a) Solve the LP relaxation with gurobi.

   (b) Apply branch and bound to find the optimal solution to the ILP.

   (c) check your solution with the help of gurobi.

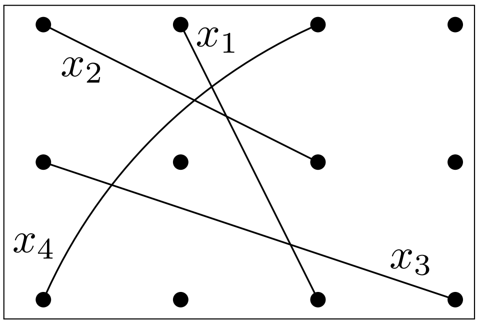
2. **Branch and Cut**
   Apply the cutting plane method to compute an optimal alignment of two sequences "'$ACCA$"' and "'$CACA$"' where a match scores 1 and a mismatch or gap scores 0:

   (a) Draw the alignment graph, the conflict graph, and the pair graph.

   (b) Now start with the trivial (relaxed) LP and add successively clique inequalities which you can find on the longest paths in the pair graph that is labeled with the solution of the last step. Repeat this until you get the optimal alignment.

3. **Branch and Cut**

Given the following alignment graph:



All edges have weight 1.

(a) Try to solve the alignment problem by using branch-and-cut: Add mixed cycle inequalities (see the 'shortest path' method in the script, page 18) to the corresponding (relaxed) LP. Can you reach an optimal solution for the ILP without branching?

(b) Now use branching to solve the problem.

(c) Instead of branching, just add the inequality

$$x_1 + x_2 + x_3 + x_4 \leq 2$$

Can you solve the ILP now?

(d) Prove that the inequality in (c) is facet-defining.

4. **MILP**

Given a metabolic network: a stoichiometric matrix $S \in \mathbb{R}^{m \times n}$, the set of indices corresponding to the irreversible reactions Irr, and lower and upper bounds for the fluxes of the reactions, lb and ub respectively.

Build a Mixed Integer Linear Program (MILP) such that:

- the system is in steady state
- the fluxes respect the lower and upper bounds
- the number of active reactions (reactions which carry flux) is minimal
- the trivial flux vector ($v = 0$) is not a feasible solution

Thus we search for a flux vector v s.t. the number of entries with $v_i \neq 0$ is minimal.

(a) suppose that all reactions are irreversible.

(b) there exists reversible reactions.

(*Hint:* To minimise over the number of active reactions, binary variables are needed. For each v_i there has to exist a binary a_i which indicates that v_i carries flux or not. Thus: $a_i = 0 \;\Leftrightarrow\; v_i = 0$. Try to formulate "$\Leftrightarrow$" with the help of linear constraints.)