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January 15, 2016

Deadline: January 14, 2016, 11:45 am

Optimization

WS 2015/16

Exercises 3

1. **Stable marriage problem**
   Solve the *Stable-marriage-problem* with the greedy algorithm discussed in the lecture: Given are 5 men and women, all of them straight. The task is to marry them together where polygamy is not allowed.
   The men ranked the women in order of preferences and so the women did. Their ranks are given by these matrices:

$$\begin{array}{c} \\ w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{array} \begin{array}{c} \begin{array}{ccccc} r_1 & r_2 & r_3 & r_4 & r_5 \end{array} \\ \begin{pmatrix} 2 & 4 & 5 & 1 & 3 \\ 1 & 3 & 5 & 2 & 4 \\ 2 & 1 & 4 & 3 & 5 \\ 5 & 4 & 1 & 2 & 3 \\ 3 & 1 & 2 & 5 & 4 \end{pmatrix} \end{array} \qquad \begin{array}{c} \\ m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \end{array} \begin{array}{c} \begin{array}{ccccc} r_1 & r_2 & r_3 & r_4 & r_5 \end{array} \\ \begin{pmatrix} 1 & 3 & 2 & 5 & 4 \\ 1 & 2 & 5 & 4 & 3 \\ 4 & 1 & 5 & 3 & 2 \\ 3 & 2 & 4 & 5 & 1 \\ 5 & 4 & 1 & 2 & 3 \end{pmatrix} \end{array}$$

   w_i denotes the i-th woman and m_j the j-th man. In the first column are the most preferable persons. Thus woman 1 prefers at most man 2. After man 2 she ranked man 4 and so on.
   Write down the steps of the method and the optimal solution.

2. **Critical Mixed Cycles**
   Prove the following lemma (see lecture script):

   A subset $T \subseteq E$ is a trace, if and only if $G' = (V, T, H)$ does not contain a critical mixed cycle.

3. **n-Queens-Problem**

   Write down an ILP for the so called *n-queens-problem*:
   Place as much queens as possible on a $n \times n$ chess board such that no two queens interfere. Thus:

   - In each vertical line ...
   - In each horizontal line ...
   - In each diagonal line ...

   ... is only one queen allowed