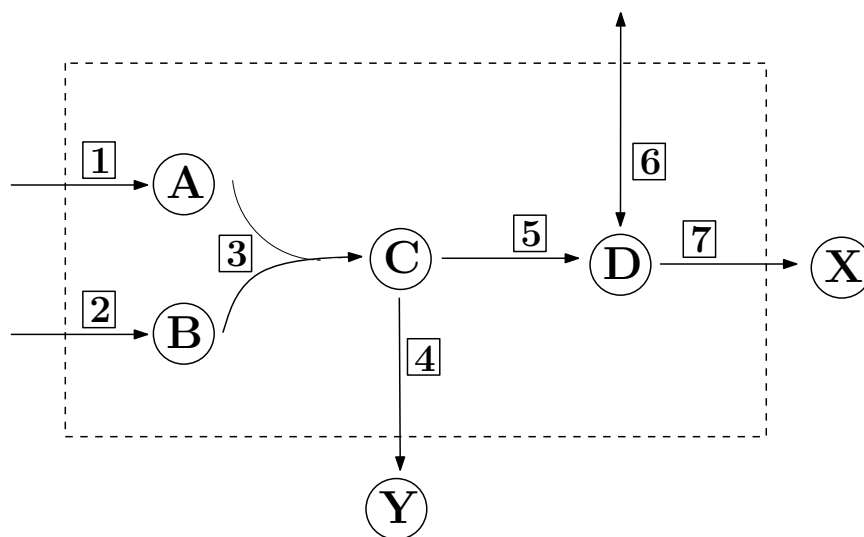


# Optimization

WS 2015/16

## Exercises 2

1. Find the stoichiometric matrix of the following network:  
(Hint: The stoichiometric matrix only has rows corresponding to internal metabolites.)



Reaction 1 is allowed to carry a flux of at most 1.

- (a) Maximize the production of metabolite  $Y$ . Write down the LP-Form and the dual of it.
- (b) Maximize the production of metabolite  $X$ . Write down the LP-Form and the dual of it.

Solve the problems of a) and b) with Gurobi (the primal as well as the dual) and analyse your results.

2. Consider the polyhedron  $P \subset \mathbb{R}^2$  defined by the system of linear inequalities

$$x_1 + x_2 \leq 5, \quad 0 \leq x_1 \leq 3, \quad 0 \leq x_2 \leq 2 \quad (*)$$

- (a) Write  $(*)$  in the form  $Ax \leq b$ , for some  $A \in \mathbb{R}^{m \times n}$  and  $b \in \mathbb{R}^m$ .
- (b) Determine the bases of  $(*)$  and the corresponding basic solutions.
- (c) Which of them are feasible?
- (d) Give for each vertex of  $P$  the corresponding feasible bases.

3. Consider the linear program  $\max\{c^T x \mid Ax \leq b\}$ , where

$$A = \begin{pmatrix} -2 & 3 \\ 2 & 3 \\ 1 & 0 \\ -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad c = \begin{pmatrix} 1 \\ 4 \end{pmatrix}, \quad b = \begin{pmatrix} 6 \\ 12 \\ 3 \\ 0 \\ 0 \end{pmatrix}.$$

A feasible basis is given by  $I = \{3, 5\}$  with the corresponding basic feasible solution  $v = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ .

Solve the LP using the simplex algorithm: start at the given vertex. Use the pivoting rules and do as many iterations as you need until you find the maximum. Prove that you found the maximum using the theory for the simplex.

Write down your steps in detail (direction  $d$ , largest  $\lambda$ , exchange of indices).

4. Consider the linear optimization problem:

$$\begin{aligned} \max \quad & c_1 x_1 + c_2 x_2 \\ \text{w.r.t.} \quad & 2x_1 - x_2 \leq 2 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Determine coefficients  $(c_1, c_2)$  of the objective function such that

- (a) the problem has a unique optimal solution.
- (b) the problem has multiple optimal solutions and the set of optimal solutions is bounded.
- (c) the problem has multiple optimal solutions and the set of optimal solutions is unbounded.
- (d) the problem has feasible solutions, but no optimal solutions.