

## Case study: Time tabling

Rossi-Doria et al. 2002 [http://dx.doi.org/10.1007/978-3-540-45157-0\\_22](http://dx.doi.org/10.1007/978-3-540-45157-0_22)

- Set of events  $E$ , set of rooms  $R$ , set of students  $S$ , set of features  $F$
- Each student attends a number of events and each room has a size.
- Assign all events a timeslot and a room so that the following *hard constraints* are satisfied:
  - no student attends more than one event at the same time.
  - the room is big enough for all attending students and satisfies all features required by the event.
  - only one event is in each room at any timeslot.

## Case study: Time tabling <sup>(2)</sup>

- Penalties for *soft constraint* violations
  - a student has a class in the last slot of a day.
  - a student has more than two classes in a row.
  - a student has a single class on a day.
- Objective: Minimize number of soft constraint violations in a feasible solution

## Common neighborhood structure

- *Solution*  $\rightsquigarrow$  ordered list of length  $|E|$   
The  $i$ -th element indicates the timeslot to which event  $i$  is assigned.
- Room assignments generated by matching algorithm.
- *Neighborhood*:  $N = N_1 \cup N_2$ 
  - $N_1$  moves a single event to a different timeslot
  - $N_2$  swaps the timeslots of two events.

## Common local search procedure

*Stochastic first improvement local search*

- Go through the list of all the events in a random order.
- Try all the possible moves in the neighbourhood for every event involved in constraint violations, until improvement is found.
- Solve hard constraint violations first.  
If feasibility is reached, look at soft constraint violations as well.

## Metaheuristics

1. Evolutionary algorithm
2. Ant colony optimization
3. Iterated local search

- 4. Simulated annealing
- 5. Tabu search

## 1. Evolutionary algorithm

- *Steady-state evolution process*: at each generation only one couple of parent individuals is selected for reproduction.
- *Tournament selection*: choose randomly a number of individuals from the current population and select the best ones in terms of fitness function as parents.
- *Fitness function*: Weighted sum of hard and soft constraint violations,

$$f(s) := \#hcv(s) \cdot C + \#scv(s)$$

## 1. Evolutionary algorithm <sup>(2)</sup>

- *Uniform crossover*: for each event a timeslot's assignment is inherited from the first or second parent with equal probability.
- *Mutation*: Random move in an extended neighbourhood (3-cycle permutation).
- *Search parameters*: Population size  $n = 10$ , tournament size = 5, crossover rate  $\alpha = 0.8$ , mutation rate  $\beta = 0.5$
- Find a balance between the number of steps in local search and the number of generations.

## 2. Ant colony optimization

- At each iteration, *each of  $m$  ants constructs*, event by event, a *complete assignment* of the events to the timeslots.
- To make an assignment, an ant takes the next event from a pre-ordered list, and probabilistically chooses a timeslot, guided by two types of information:
  1. *heuristic information*: evaluation of the constraint violations caused by making the assignment, given the assignments already made,
  2. *pheromone information*: estimate of the utility of making the assignment, as judged by previous iterations of the algorithm.
- *Matrix* of pheromone values  $\tau : E \times T \rightarrow \mathbb{R}_{\geq 0}$ .  
Initialization to a parameter  $\tau_0$ , update by local and global rules.

## 2. Ant colony optimization <sup>(2)</sup>

- An event-timeslot pair which has been part of good solutions will have a high pheromone value, and consequently have a higher chance of being chosen again.
- At the end of the iterative construction, an event-timeslot assignment is converted into a candidate solution (timetable) using the matching algorithm.
- This candidate solution is further improved by the local search routine.
- After all  $m$  ants have generated their candidate solution, a global update on the pheromone values is performed using the best solution found since the beginning.

### 3. Iterated local search

- Provide new starting solutions obtained from **perturbations** of a current solution
- Often leads to far better results than using random restart.
- Four subprocedures
  1. *GenerateInitialSolution*: generates an initial solution  $s_0$
  2. *Perturbation*: modifies the current solution  $s$  leading to some intermediate solution  $s'$ ,
  3. *LocalSearch*: obtains an improved solution  $s''$ ,
  4. *AcceptanceCriterion*: decides to which solution the next perturbation is applied.

#### Perturbation

- *Three types of moves*
  - P1**: choose a different timeslot for a randomly chosen event;
  - P2**: swap the timeslots of two randomly chosen events;
  - P3**: choose randomly between the two previous types of moves and a 3-exchange move of timeslots of three randomly chosen events.
- *Strategy*
  - Apply each of these different moves  $k$  times, where  $k$  is chosen of the set  $\{1; 5; 10; 25; 50; 100\}$ .
  - Take random choices according to a uniform distribution.

#### Acceptance criteria

- Random walk: Always accept solution returned by local search
- Accept if better
- Simulated annealing

$$\text{SA1: } P_1(s, s') = e^{-\frac{f(s) - f(s')}{T}}$$

$$\text{SA2: } P_2(s, s') = e^{-\frac{f(s) - f(s')}{T \cdot f(s_{\text{best}})}}$$

*Best parameter setting* (for medium instances):

**P1**,  $k = 5$ , **SA1** with  $T = 0.1$

### 4. Simulated annealing

*Two phases*

1. Search for feasible solutions, i.e., satisfy all hard constraints.
2. Minimize soft constraint violations.

#### Strategies

- *Initial temperature*: Sample the neighbourhood of a randomly generated solution, compute average value of the variation in the evaluation function, and multiply this value by a given factor.

- *Cooling schedule*

1. Geometric cooling:  $T_{n+1} = \alpha \times T_n$ ,  $0 < \alpha < 1$

2. Temperature reheating: Increase temperature if *rejection ratio* (= number of moves rejected/number of moves tested) exceeds a given limit.

- *Temperature length* (number of iterations at each temperature): Proportional to the size of the neighborhood

## 5. Tabu search

- Moves done by moving one event or by swapping two events.
- Explore solutions that do not decrease the objective function value
- *Tabu list*: Forbid a move if at least one of the events involved has been moved less than  $l$  steps before.
- *Size of tabu list  $l$* : number of events divided by a suitable constant  $k$  (here  $k = 100$ ).
- *Variable neighbourhood set*: every move is a neighbour with probability 0.1  $\rightsquigarrow$  decrease probability of generating cycles and reduce the size of neighbourhood for faster exploration.
- *Aspiration criterion*: perform a tabu move if it improves the best known solution.

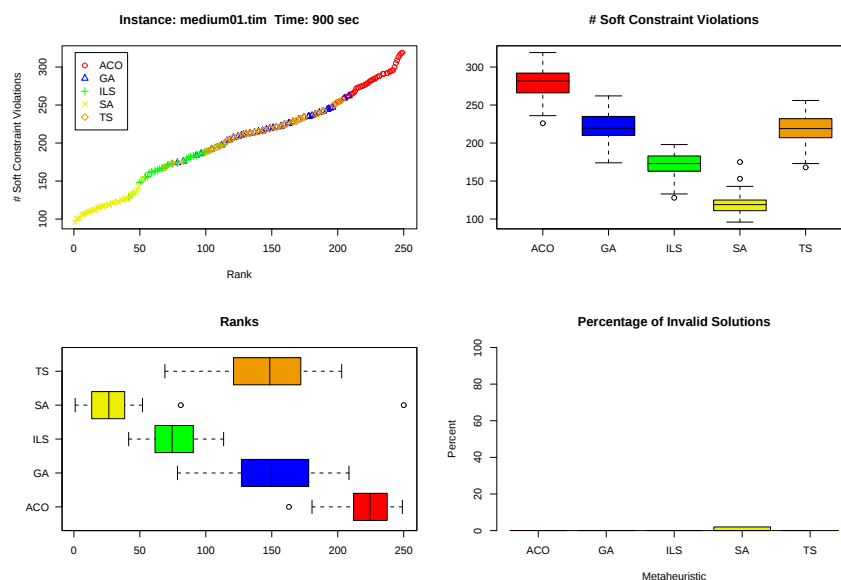
## Evaluation

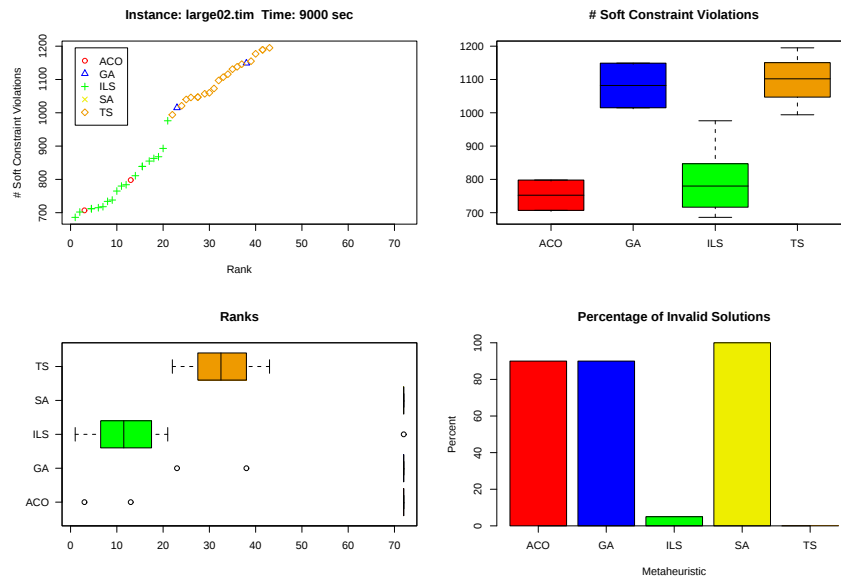
<http://iridia.ulb.ac.be/~msampels/ttmn.data/>

- 5 small, 5 medium, 2 large instances

| Type  | small | medium | large |
|-------|-------|--------|-------|
| $ E $ | 100   | 400    | 400   |
| $ S $ | 80    | 200    | 400   |
| $ R $ | 5     | 10     | 10    |

- 500 resp. 50 resp. 20 independent trials per metaheuristic per instance.
- Diagrams show results of all trials on a single instance.
- Boxes show the range between 25% and 75% quantile.





## Evaluation (2)

- *Small*: All algorithms reach feasibility in every run, ILS best, TS worst overall performance
- *Medium*: SA best, but does not achieve feasibility in some runs. ACO worst.
- *Large01*: Most metaheuristics do not even achieve feasibility. TS feasibility in about 8% of the trials.
- *Large02*: ILS best, feasibility in about 97% of the trials, against 10% for ACO and GA. SA never reaches feasibility. TS gives always feasible solutions, but with worse results than ILS and ACO in terms of soft constraints.