Network Analysis SS 17
Continuous Modeling

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Deadline: Monday, 15 May, 12:00 am

1 Exercise

Consider the system of differential equations

$$\begin{aligned}\dot{x} &= x^2 - y \\ \dot{y} &= x + y\end{aligned}$$

1. Determine the nullclines and the critical points.
2. For each critical point, determine the Jacobi matrix and its eigenvalues.
3. What does this imply for the type of the critical points?
4. Sketch a phase portrait and check using `XPP`.

2 Exercise

Consider the system of differential equations

$$\begin{aligned}\dot{x} &= x^2 - 1 \\ \dot{y} &= 2y\end{aligned}$$

1. Determine the nullclines and the critical points.
2. For each critical point, determine the Jacobi matrix and its eigenvalues.
3. What does this imply for the type of the critical points?
4. Sketch a phase portrait and check using `XPP`.

3 Exercise

Consider the system of ordinary differential equations

$$\begin{aligned}\dot{x} &= -ax + y \\ \dot{y} &= \frac{x^2}{1+x^2} - by\end{aligned}$$

where $a, b > 0$ and for $x, y \in \mathbb{R}_{\geq 0}$.

1. Sketch the nullclines (in a $x - y$ coordinate system).
2. Suppose b is fixed and a varies. How many critical points can be observed for small values of a? How many for big values?
3. Compute the intersection points of the nullclines. How many (real) solutions can we get and how does that depend on a and b?
4. For fixed b compute the value a_c, where the system changes its behaviour. What kind of bifurcation point is a_c?
5. Analyse the stability of the critical points. You can use `XPPAUT`. Start with the following input and then vary the parameter a. Draw the nullclines and the direction field and derive the stability of the critical points for different values of a.

```
#Exercise 3, Bifurcations
par a=.5,b=.5
x' =-a*x+y
y' =x^2/(1+x\^2)-b*y
done
```

6. Draw different phase planes, one for $a < a_c$ and one for $a > a_c$.

4 Exercise

Consider the transcritical bifurcation

$$\dot{x} = rx + x^2$$

1. Determine the critical points.
2. Draw the critical points into a bifurcation diagram.
3. Discuss the stability of the critical points.
4. Complete the bifurcation diagram by adding trajectories for x.

5. What happens at a transcritical bifurcation point with the dynamics of the system?

Send the solutions for exercise 2, 3.4, 3.5, 4.2, 4.4, and 4.5 until Monday 15. May, 12:00 am to `Annika.Roehl@fu-berlin.de`