Tutorial Network Analysis

Freie Universität Berlin, SS 2016/17
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Assignment 1
Due date: 26.6.2017 10:00 AM before the lecture

Include all important steps of your calculations/solutions. Give the important parts of your code or send the complete code to: `alena.vanboemmel@molgen.mpg.de`. Build groups of max. 2 students to solve the problems.

Name(s): Matrikelnr.:

Problem 1 (*30 Points; Gene Networks*)**.** Consider the following RPKM values from 5 RNA-seq experiments for the following genes:

Gene	Exp1	Exp2	Exp3	Exp4	Exp5
A	0.5	1.3	3.3	4.3	5.7
B	5.8	6.9	4.5	1.3	1.8
C	7.8	10.0	15.6	20.9	35.6
D	8.6	7.0	6.5	7.1	8.7
E	18.8	14.7	7.5	15.1	18.2

Draw the gene network with the following criteria for edges:

(A) Draw an edge between genes X and Y if the Euclidean distance:

$$d_E(X,Y) := \sqrt{\sum_{i=1}^{n}(x_i - y_i)^2} < 12$$

. n is the number of samples (i.e. experiments) and $X, Y \in \{A, B, C, D, E\}$.

(B) Draw an edge between genes X and Y if the correlation coefficient $|r(X,Y)| > 0.8$. Color the edges with positive correlation red and the edges with negative correlation blue.

(C) Draw an edge between genes X and Y if the L_1-norm:

$$||X,Y||_{L_1} := \frac{1}{n}\sum_{i=1}^{n}|x_i - y_i| < 8$$

.

(D) Draw an edge between genes X and Y if the mutual information:

$$I(X,Y) := \sum_{x\in X}\sum_{y\in Y} p(x,y)\log(\frac{p(x,y)}{p(x)p(y)}) > 0.6.$$

To calculate the mutual information, you bin the RPKM values for each gene into 3 intervals.

Hint: You can use the R package `infotheo` *for binning and calculation of the mutual information.*

Problem 2 (*40 Points; Probabilistic Distribution, Independence, Information Theory*)**.** Consider two random variables X and Y from which we drew the following samples:

$$\begin{aligned} x &= (0.51, 0.99, 0.64, 0.50, 0.27, 0.12, 0.01, 0.79, 0.56, 0.17) \\ y &= (0.74, 0.06, 0.43, 0.12, 0.61, 0.73, 0.57, 0.91, 0.59, 0.80) \end{aligned}$$

First, bin the data by dividing the interval of $[0, 1]$ into 4 equally wide sub-intervals. Provide the following calculations *by hand.*

(A) Calculate the joint probability distribution $p_{X,Y}(x, y)$ of the binned data and write it in the following table:

$Y\|X$	x_1	x_2	x_3	x_4
y_1				
y_2				
y_3				
y_4				

(B) Calculate the marginal distributions $p_X(x)$ and $p_Y(y)$

(C) Calculate the product of the two marginal distributions $p_X(x) \times p_Y(y)^T$ (matrix multiplication!) and compare it with the joint distribution $p_{X,Y}(x, y)$. Are variables X and Y stochastically independent? Justify your answer.

(D) Calculate the conditional distributions $p_{X|Y}(x|y = y_3)$ and $p_{Y|X}(y|x = x_4)$

(E) Calculate the joint entropy $H(X, Y)$ and the marginal entropies $H(X)$ and $H(Y)$

(F) Calculate the conditional entropies $H(X|Y)$ and $H(Y|X)$ using the chain rule.

(G) Calculate the mutual information $I(X, Y)$ using both, the definition and the relation to entropy. Are both results equal? Why?

Problem 3 (*20 Points; Gaussian distribution*)**.** Analyze the following two cases of Gaussian distribution.

(A) Consider a two-dimensional random variable (X, Y) from a bivariate Gaussian distribution:

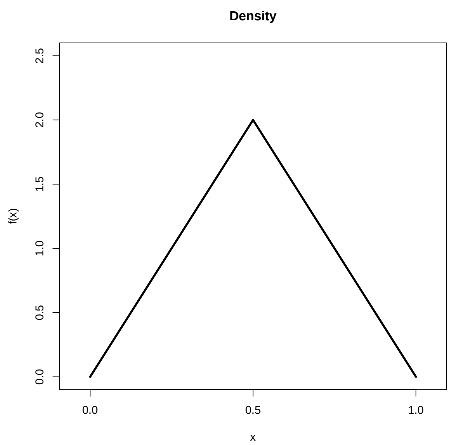
$$(X, Y) \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0.7 \\ 0.7 & 1 \end{pmatrix}\right).$$

Draw a random sample from the bivariate Gaussian distribution with the size $n_1 = 10$ and calculate the correlation coefficient of the two vectors. Repeat the step for sample size $n_2 = 100$ and $n_3 = 1000$. What do you observe? What is the relation of the correlation coefficient and the covariance matrix?

(B) Draw a random sample of size $n = 100$ from a univariate Gaussian distribution with mean $\mu = 0$ and variance $\sigma^2 = 3$. Draw another random sample of the same size from a univariate Gaussian distribution with mean $\mu = 1$ and variance $\sigma^2 = 2$. Calculate the correlation coefficient of these two vectors. Are the two variables independent?

Hint: You can use the R package `MASS` *for simulation of the multivariate Gaussian distribution. Use* `set.seed(n)` *and give the integer* `n` *that you used for your simulation.*

Problem 4 (*10 Points; Expected value*)**.** Consider the following density function $f_X(x)$ of random variable X:



Calculate the expected value EX.