Netzwerkanalyse SS 2015
Alexander Bockmayr
Therese Lorenz

Exercise Sheet 3

May 11, 2015

Exercise 1. *Tutorial*
Analyse the transcritical bifurcation

$$\dot{x} = rx + x^2$$

a) Determine the critical points.

b) Draw the critical points into a bifurcation diagram.

c) Discuss the stability of the critical points.

d) Complete the bifurcation diagram by adding trajectories for x.

e) What happens at a transcritical bifurcation point with the dynamics of the system?

Exercise 2. *Tutorial*
Consider the system of ordinary differential equations:

$$\dot{x} = -ax + y$$
$$\dot{y} = \frac{x^2}{1+x^2} - by$$

where $a, b > 0$ and for $x, y \in \mathbb{R}_{\geq 0}$.

a) Sketch the nullclines (in a $x - y$ coordinate system).

b) Suppose b is fixed and a varies. How many critical points can be observed for small values of a? How many for big values?

c) Compute the intersection points of the nullclines. How many (real) solutions can we get and how does that depend on a and b?

d) For fixed b compute the value a_c, where the system changes its behaviour. What kind of bifurcation point is a_c?

e) Analyse the stability of the critical points.
You can use XPPAUT. Start with the following input and then vary the parameter a. Draw the nullclines and the direction field and derive the stability of the critical points for different values of a.

```
#Exercise 3, Bifurcations
par a=.5,b=.5
x' =-a*x+y
y' =x^2/(1+x^2)-b*y
done
```

f) Draw different phase planes, one for $a < a_c$ and one for $a > a_c$.

Exercise 3. *Homework* 4 P.

Consider the following description of a gene regulatory network:

- $X1$ activates $X2$ synthesis at low concentrations and it represses its own synthesis at high concentrations.
- $X2$ represses the synthesis of $X1$.

Build and analyse a discrete model using the following the steps:

a) Draw the corresponding interaction graph I.

b) Give a differential equation model of I using Hill functions.

c) Give the logical equations for the corresponding discrete model.

d) What are the possible values of the discrete variables?

e) What are the logical parameters of the discrete model and what is their range?

f) Give the state table of the discrete model in terms of the logical parameters.

For the notation see slides of May, 8th, especially Chapters 4 and 5 (`https://www.mi.fu-berlin.de/wiki/pub/AgMathLife/AdditionalMaterialNetzwerk15/Discr1.pdf`).
Send the solution until Friday, 15th, 10:00 a.m. to `Therese.Lorenz@fu-berlin.de`.