Prof. Dr. Alexander Bockmayr,
Prof. Dr. Oliver Serang,
Christopher Pockrandt

November 20, 2015

Algorithms

WS 2015/16

Exercise 5
(discussed on November 27th, 2015)

Prepare yourself to present your solutions to your fellow students.

1. **Tree decomposition (Niveau I)**

   How large is the largest piece of a any tree decomposition for a graph G of n nodes if G is a clique? Prove your answer.

2. **Tree decomposition (Niveau II)**

   As discussed in the lecture the maximum-weighted independent set (WIS) problem can be solved on a graph with small tree width using its tree decomposition. Let T be the tree decomposition of such a graph G rooted at a node r. For any node $t \in T$ let T_t denote the subtree rooted at t and let G_t denote the subgraph associated with this subtree. Further let S be a maximum-weighted independent set of G_t subject to the requirement that $S \cap V_t = U$; that is $w(S) = f_t(U)$ and let S_i denote the intersection of S with the nodes of G_{t_i}.

   Prove the following theorem:
   S_i is a maximum-weighted independent set of G_{t_i}, subject to the constraint that $S_i \cap V_t = U \cap V_{t_i}$.

   Note, that $w(S) = \Sigma_{s \in S} w_s$ is the total weight of all nodes in $S \subseteq V_t$,
   and $f_t(U)$ denotes the maximum weight of an independent set S in G_t, subject to the requirement that $S \cap V_t = U$, where $U \subseteq V_t$ is an independent set.

3. **Tree decomposition (Niveau I)**

Use the algorithm presented in the lecture to compute a tree decomposition of the graph below. Use $w = 4$: