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Algorithms

WS 2015/16

Exercise 4

(discussed on November 13th, 2015)

Prepare yourself to present your solutions to your fellow students.

1. Modulo Arithmetic (Niveau II)

Prove the following theorem:

For any positive integers a and n , if $d = \gcd(a, n)$ (the greatest common divisor of a and n), then

$$\langle a \rangle = \langle d \rangle = \{0, d, 2d, \dots, n - d\}$$

and thus

$$|\langle a \rangle| = n/d$$

($\langle a \rangle := \{a \cdot i \bmod n \mid i \in \mathbb{N}\}$).

Hint: Use Bezout's lemma. It states that if a and b are nonzero integers with greatest common divisor d , then there exist integers x and y such that $ax + by = d$

2. Hashing (Niveau I)

Consider a version of the division method in which $h(k) = k \bmod m$, where $m = 2^p - 1$ and k is a character string interpreted in radix 2^p . Show that if string x can be derived from string y by permuting its characters, then x and y hash to the same value.

3. Hashing (Niveau I)

Consider the two situations in a hash table of size m using open addressing with linear probing:

- You have $n = m/2$ keys in the table, where every even-indexed slot is occupied and every odd-indexed slot is free.
- You have $n = m/2$ keys in the table and the first $n = m/2$ locations are the ones occupied.

(a) Compute the average search cost for an unsuccessful search for both situations under the hypothesis of simple uniform hashing.

4. Skip lists (Niveau I)

Compute the expected value for the height (h), search time and space consumption if the probability p for each coin flip to produce a 1 is $1/3$.