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Algorithms

WS 2015/16

Exercises 1
(discussed on October 23th, 2015)

Prepare yourself to present your solutions to your fellow students.

1. **Landau Symbols (Niveau I)**
   Prove or disprove the following:

   (a) $\forall k, l \in \mathbb{Z}.k > l : n^l \in o(n^k)$

   (b) $\forall k, l \in \mathbb{N}.k > l : n^k + n^l \in \Theta(n^k)$

   (c) $f \in O(2^n) \Leftrightarrow f \in 2^{O(n)}$

2. **MST - Approximation (Niveau I)**

   (a) Construct a complete graph with at least 6 nodes that satisfies the triangle inequality and apply the MST- approximation algorithm to approximate the optimal solution of the TSP.

   (b) Prove that the MST-approximation is a 2-approximation for the TSP.

3. **Amortized Analysis (Niveau I)**
   Assume an array of a certain initial size n. After n insertions the array is full and to insert more elements the array needs to be resized. One approach is to allocate a bigger array and to copy all previously inserted elements into the new array. The cost for insertion and copy of an element is O(1) each.

   (a) How would you choose the size of the new array if you have to allocate additional space to achieve amortized constant runtime for inserting an element?

   (b) Use the potential method and accounting method to show that the amortized cost is indeed constant.

4. **Analysis of SELECTION algorithm (Niveau II)**
   Read the additional PDF document (AdditionalMaterial) about selecting the k-th smallest element in worst-case linear time and solve Exercise 9.3-1