1 Exercise

Consider the gene interaction network $\mathcal{I}$:

Assume that upon activation, $X_1$ first acts on $X_2$, and then on itself.

1. What is the informal meaning of the two CTL formulas $EF(X_1 = 2 \land X_2 = 1)$ resp. $AF(X_1 = 2 \land X_2 = 1)$?

2. For each of these two formulas, give the set of states in $\mathcal{T}$ for which it is true.

3. For each of the CTL formulas $AF(X_2 = 1)$, $AG(X_2 = 1)$, and $EX(AG(X_2 = 1))$, give the set of states in $\mathcal{T}$ for which it is true.
2 Exercise

1. Given the state transition graph $G$

use the labeling algorithm to determine all states of $G$ in which the formula

- $AF(E[p U q] \land EXp)$
- $AF EG (p \land q)$

is true.

2. Represent the following properties by CTL formula:

- $q$ is never true after $p$.
- $p$ is never true between $q$ and $r$. 
3 Exercise

Which of the specifications below convey the mathematical meaning of the CTL formula $AG (p \rightarrow A[q U r])$ ?

1. Any reachable state in which $p$ is true has a path from it on which $r$ is eventually true, and until then $q$ is true.

2. If $p$ is true in every reachable state, then there is a path along which $q$ is continuously true, until $r$ becomes true.

3. If $p$ is true in every reachable state, then for any path along which $q$ is continuously true, $r$ becomes true.

4. For any reachable state in which $p$ is true, then, on any path from that state, $q$ is continuously true until $r$ becomes true, and $r$ is guaranteed to become true.

5. If $p$ is true in every reachable state, then on every path there is a state at which $r$ is true, and $q$ is true continuously until then.

4 Exercise

Which of the following pairs of CTL formulas are equivalent ?

1. $EF p$ and $EG p$

2. $EF p \lor EF q$ and $EF (p \lor q)$

3. $AF p \lor AF q$ and $AF (p \lor q)$

4. $AF p$ and $A[p U true]$

5. $EF \neg p$ and $\neg AF p$

5 Exercise

Consider the transition system $(S, \rightarrow, L)$ where,

- the set of states $S = \{s_0, s_1, s_2, s_3\}$,
- the state transitions are $(s_0, s_0), (s_0, s_1), (s_0, s_3), (s_1, s_2), (s_2, s_1), (s_3, s_2)$,
- the labeling function is given by $L(s_0) = \{r\}, L(s_1) = \{p, r\}, L(s_2) = \{q, r\}$ and $L(s_3) = \{p, q\}$.
Which of the CTL formulas below are satisfied in state $s_0$?

1. $AF(q \land r)$
2. $AG(p \rightarrow AF(p \land r))$
3. $A[rUq]$
4. $AG(p \rightarrow AG(p \lor q))$
5. $AG EF\neg r$