# Network Analysis SS 14
# Discrete Modeling

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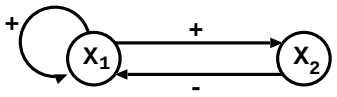
## 1 Exercise

Consider the gene interaction network $\mathcal{I}$:

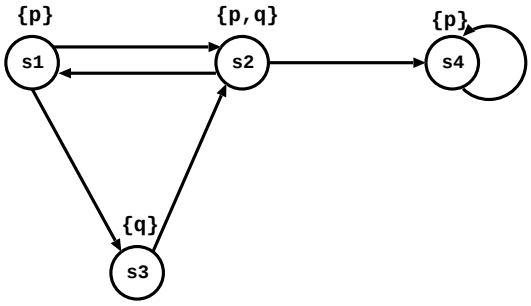



Assume that upon activation, $X_1$ first acts on $X_2$, and then on itself.

1. What is the informal meaning of the two CTL formulas $EF(X_1 = 2 \wedge X_2 = 1)$ resp. $AF(X_1 = 2 \wedge X_2 = 1)$?
2. For each of these two formulas, give the set of states in $\mathcal{T}$ for which it is true.
3. For each of the CTL formulas $AF(X_2 = 1)$, $AG(X_2 = 1)$, and $EX(AG(X_2 = 1))$, give the set of states in $\mathcal{T}$ for which it is true.

## 2 Exercise

1. Given the state transition graph $G$




   use the labeling algorithm to determine all states of $G$ in which the formula

   - $AF\,(E[p\,U\,q] \wedge EXp)$
   - $AF\,EG\,(p \wedge q)$

   is true.

2. Represent the following properties by CTL formula:

   - $q$ is never true after $p$.
   - $p$ is never true between $q$ and $r$.

## 3 Exercise

Which of the specifications below convey the mathematical meaning of the CTL formula $AG\,(p \rightarrow A[q\,U\,r])$ ?

1. Any reachable state in which p is true has a path from it on which r is eventually true, and until then q is true.
2. If p is true in every reachable state, then there is a path along which q is continuously true, until r becomes true.
3. If p is true in every reachable state, then for any path along which q is continuously true, r becomes true.
4. For any reachable state in which p is true, then, on any path from that state, q is continuously true until r becomes true, and r is guaranteed to become true.
5. If p is true in every reachable state, then on every path there is a state at which r is true, and q is true continuously until then.

## 4 Exercise

Which of the following pairs of CTL formulas are equivalent ?

1. $EF\,p$ and $EG\,p$
2. $EF\,p \;\vee\; EF\,q$ and $EF\,(p \vee q)$
3. $AF\,p \;\vee\; AFq$ and $AF\,(p \vee q)$
4. $AF\,p$ and $A[p\,U\,\text{true}]$
5. $EF\,\neg p$ and $\neg AF\,p$

## 5 Exercise

Consider the transition system $(S, \rightarrow, L)$ where,

- the set of states $S = \{s_0, s_1, s_2, s_3\}$,
- the state transitions are $(s_0, s_0), (s_0, s_1), (s_0, s_3), (s_1, s_2), (s_2, s_1), (s_3, s_2)$,
- the labeling function is given by $L(s_0) = \{r\}, L(s_1) = \{p, r\}, L(s_2) = \{q, r\}$ and $L(s_3) = \{p, q\}$.

Which of the CTL formulas below are satisfied in state $s_0$ ?

1. $AF(q \wedge r)$
2. $AG(p \rightarrow AF(p \wedge r))$
3. $A[rUq]$
4. $AG(p \rightarrow AG(p \vee q))$
5. $AG\ EF\neg r$