Networkanalysis SS 14
Dynamic Modeling – Exercises II

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1 Exercise

Consider the system of ordinary differential equations:

$$\dot{x} = -ax + y \tag{1}$$

$$\dot{y} = \frac{x^2}{1+x^2} - by \tag{2}$$

where $a, b > 0$.

- Show that this system has three fixed points when $a < a_c$, where a_c is to be determined.
- Show that two of these fixed points coalesce in a saddle-node bifurcation when $a = a_c$.

Hints for solution:

1. Sketch the nullclines of this system.
2. Suppose we vary a while holding b fixed. How many steady states can be observed for a small? How many when a is large?
3. Compute the intersection points, and determine a_c.

Check your results with XPPAUT. You may use the following input file:

```
#Exercise 1, Bifurcations
par a=.5,b=.5
x' =-a*x+y
y' =x^2/(1+x^2)-b*y
done
```

- Draw the nullclines and direction field.

- Analyse the stability of the steady states.
- Change the value of the parameter a and draw nullclines for the different values.
- What kind of steady states can be observed for small a ?

Biological interpretation: Think about a gene with an autocatalytic feedback loop. The variables x and y represent the concentrations of the protein and the mRNA from which it is translated. The system can act like a biochemical switch, but only if the mRNA and protein degrade slowly enough, i.e., if their decay rates satisfy $ab < 1/2$. In this case there are two stable steady states: one at the origin, meaning that the gene is silent and there is no protein around to turn it on; and one where x and y are large, meaning that the gene is active and sustained by the level of the protein. The behavior is determined by the initial values of x and y.

- In XPPAUT draw a bifurcation diagram for the system (1-2) with $a = b = 0.5$.
    1. Integrate the system to reach the non-trivial stable steady state.
    2. Use AUTO (Menu FILE in XPPAUT) to draw a bifurcation diagram varying a from 0 to 2.
    3. Interprete the results.

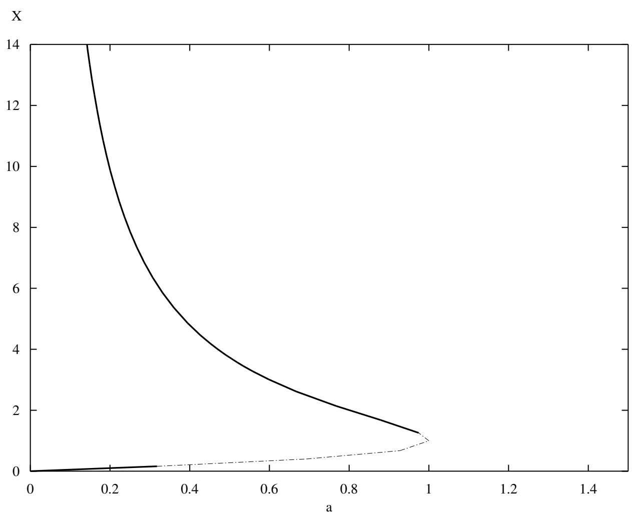


Figure 1: Strogatz SH. Nonlinear Dynamics and Chaos, Westview Press, 2001, Example 8.1.1, p. 243–245

2 Exercise

Consider the system

$$\dot{x} = \mu x - y + xy^2 \quad (3)$$
$$\dot{y} = x + \mu y + y^3 \quad (4)$$

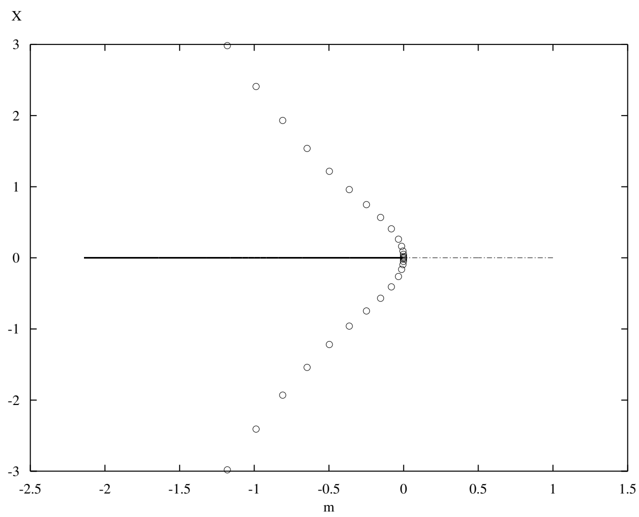
Show that a Hopf bifurcation occurs at the origin as the parameter μ varies. Hints for solution:

1. Compute the Jacobi matrix at the origin.
2. Compute the eigenvalues of this Jacobi matrix. What happens for μ equal to, smaller or greater than 0?

Use XPPAUT to draw a bifurcation diagram.

```
#Exercise 2, Hopf bifurcation
par m=-1
dx/dt=m*x-y+x*y^2
dy/dt=x+m*y+y^3
done
```

Hint: At the beginning it is the same procedure as it was for the 1. exercise. But when you finished your bifurcation then you have to grab the



point where the stability of the fixpoints is changed. After this you have to click RUN again. Then there should appear Hopf. If so you just need to click on return. If not you did something wrong.