Importance

- Biology
  - Cell metabolism
  - Catabolism, anabolism
- Medicine
  - Metabolic disorders
  - Cancer
- Biotechnology
  - Biofuel
  - Bioleaching

Mathematical representation

- Internal metabolite
- Reversible reaction
- Internal reaction
- Exchange reaction
- Irreversible reaction
- System boundary
- External metabolite
**Algebraic description**

- **Stoichiometric matrix**
  - Rows $\sim$ internal metabolites $i = 1, \ldots, m$
  - Columns $\sim$ internal and exchange reactions $j = 1, \ldots, n$
  - $S_{ij}$: stoichiometric coefficient of reactant $i$ in reaction $j$
- **Set of irreversible reactions** $\text{Irr}$
- **Metabolic model** $\mathcal{M} = (S, \text{Irr})$

**1. Kinetic modeling**

- Metabolites $i$ and reactions $j$
- $C_i(t)$: metabolite concentrations at time $t$
- $v_j(t) = v_j(C(t), k)$: reaction rates, depending on kinetic law and kinetic parameters $k$
- $S_{ij}$: stoichiometric coefficient

\[
\frac{dC_i}{dt} = \sum_{j=1}^{n} S_{ij} v_j \quad \text{or} \quad \frac{dC}{dt} = S \cdot v(C, k)
\]

- **System of ordinary differential equations (ODEs)**

**Example**

- **Steady-state assumption**
  - Assume metabolite concentrations $C_i$ and reaction rates $v_j$ are constant (over some time interval)
  - $\sim$ steady-state flux vector $v \in \mathbb{R}^n$
- **Stoichiometric constraints** (mass balance):
  \[
  \sum_{j=1}^{n} S_{ij} v_j = 0, \text{ for all } i = 1, \ldots, m
  \]
- **Thermodynamic irreversibility constraints:**
  \[
  v_j \geq 0, \text{ if } j \text{ is irreversible}
  \]

- $\sim$ system of linear equations and inequalities in $\mathbb{R}^n$

**2. Constraint-based modeling**

- **Steady-state assumption**
- **Stoichiometric constraints** (mass balance):
- **Thermodynamic irreversibility constraints:**
  - $v_j \geq 0$, if $j$ is irreversible