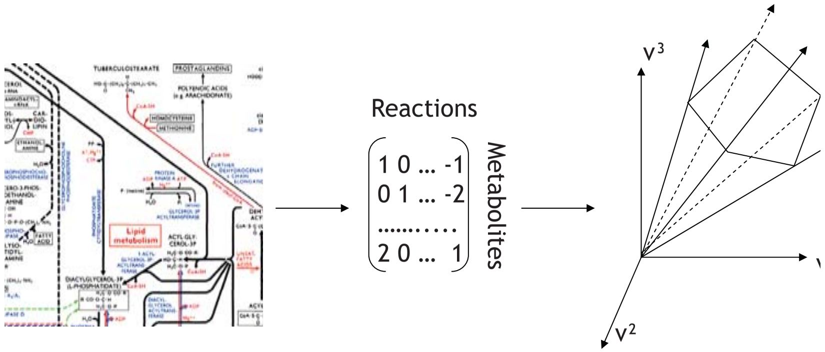
# Steady-state flux cone

Set of all possible steady-state flux distributions

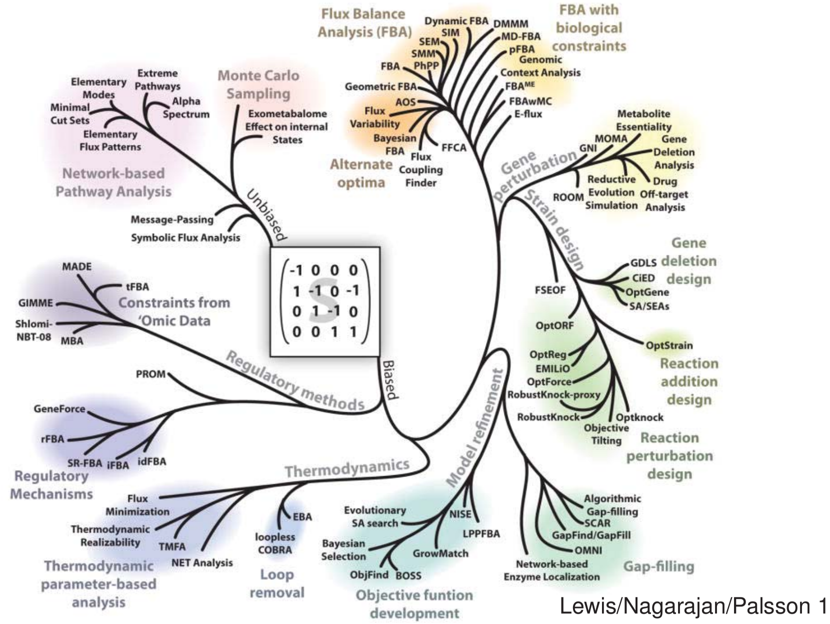
$$C = \{v \in \mathbb{R}^n \mid Sv = 0,\ v_i \geq 0,\ i \in Irr\}$$

$\rightsquigarrow$ polyhedral cone






# Constraint-based analysis methods




Lewis/Nagarajan/Palsson 12



# 3. Flux balance analysis (FBA)

- ▷ Assume cellular behavior is determined by a certain biological objective.
- ▷ Determine a corresponding “best” flux distribution.
- ▷ Use mathematical optimization to predict phenotype.
- ▷ Simplest case: Linear programming (LP)

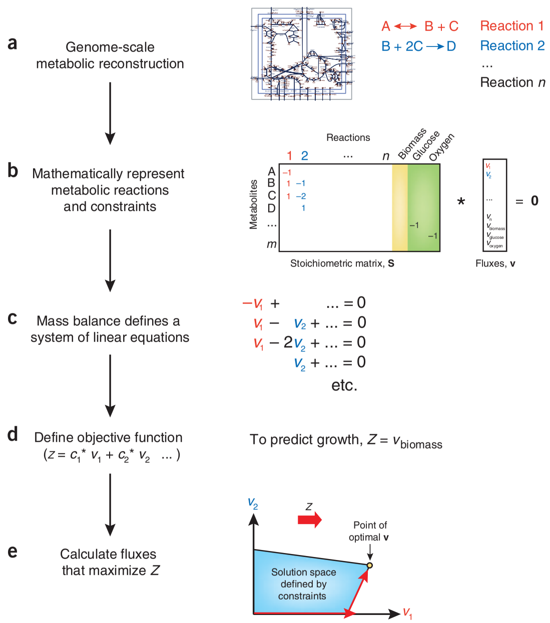
$$\max\{c^T x \mid Ax \leq b, x \in \mathbb{R}^n\}$$

- ▷ Flux balance problem (FBA)

$$\max\{c^T v \mid Sv = 0,\ l \leq v \leq u\} \qquad \text{(FBA)}$$



# Example




- ▷ *E. coli* metabolism
- ▷ Genome-scale reconstruction (*i*JO1366)
- ▷ 1336 metabolites, 2251 reactions
- ▷ Objective function: biomass
- ▷ Glucose and oxygen uptake reactions
- ▷ Aerobic and anaerobic growth
- ▷ Software: e.g. COBRA Toolbox 2.0

Orth/Thiele/Palsson 10

# 4. Flux variability analysis (FVA)

- Optimal solutions to FBA problems need not be unique.
- Enumerating all optimal solutions is computationally expensive.

- Alternative: Flux variability analysis (FVA)

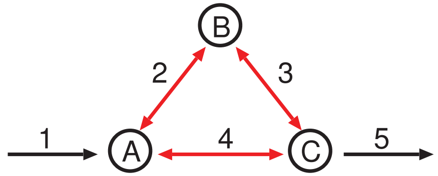
$$z_{opt} = \max\{z = c^T v \mid Sv = 0,\ l \le v \le u\} \qquad \text{(FBA)}$$

For all $j = 1, \ldots, n$:

$$\max\{\pm v_j \mid Sv = 0,\ l \le v \le u,\ c^T v = z_{opt}\} \qquad \text{(FVA)}$$



# Thermodynamic constraints




$$\begin{aligned} 1 = v_1 &= v_2 + v_4 \\ v_2 &= v_3 \\ v_3 + v_4 &= v_5 \end{aligned} \quad \Leftrightarrow \quad \begin{aligned} &v_2 \text{ free} \\ &v_4 = 1 - v_2 \\ &v_5 = 1 \end{aligned}$$

- Flux through internal cycles is unbounded $\rightsquigarrow$ thermodynamically infeasible
- Include additional (non-linear) thermodynamic constraints (Beard/Qian 02)
- Fast thermodynamic FVA (tFVA) (Müller/Bockmayr 12)
- Application: Modules in the optimal flux space (Müller/Bockmayr 13)



# 5. Flux coupling analysis (FCA)

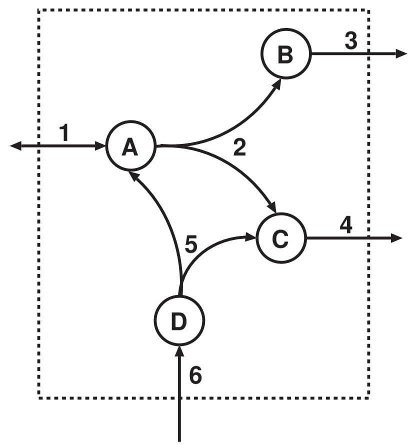
Burgard et al. 04

- $C = \{v \mid Sv = 0,\ v_k \ge 0, k \in Irr\}$ flux cone
- A reaction $i$ is blocked if $v_i = 0$, for all $v \in C$.

- Let $i$ and $j$ be two unblocked reactions.
  - $i$ is directionally coupled to $j$, $i \overset{=0}{\rightarrow} j$, if for all $v \in C$, $v_i = 0$ implies $v_j = 0$.
  - $i$ and $j$ are partially coupled, $i \overset{=0}{\leftrightarrow} j$, if for all $v \in C$, $v_i = 0$ is equivalent to $v_j = 0$.
  - $i$ and $j$ are fully coupled, $i \backsim^{\lambda} j$, if there exists $\lambda \in \mathbb{R} \setminus \{0\}$ such that for all $v \in C, v_j = \lambda\, v_i$.
- $i \backsim^{\lambda} j$ implies $i \overset{=0}{\leftrightarrow} j$, which is equivalent to $i \overset{=0}{\rightarrow} j$ and $j \overset{=0}{\rightarrow} i$.



# Example




$2 \overset{=0}{\rightarrow} 3$

$3 \overset{=0}{\rightarrow} 2$

$4 \overset{=0}{\rightarrow} 1, 2, 3, 5, 6$

$5 \overset{=0}{\rightarrow} 6$

$6 \overset{=0}{\rightarrow} 5$