# Proteomics Seminar 2014SS 

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# A linear programming model for protein inference problem in shotgun proteomics 

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## Part I

## Protein Inference Problem

## Protein inference in shotgun proteomics experiment



Figure: Protein inference using mass spectromery data. ${ }^{\mathbf{1}}$

Goal: Find a subset of proteins that are truly present in the sample.

[^0]
## What input do we need for protein inference?

Protein Peptide


- A list of identified peptides.

1. Database-driven approach
2. de novo algorithm

- Peptide probabilities (detecbilities). <- rigorous statitical validation PeptideProphet ${ }^{2}$ estimates $\operatorname{Pr}(+\mid S)$ : the probability that the peptide assignment with discriminant score S is correct.
- A list of candidate proteins.
- Expected output: a set of proteins accompanying protein probabilities.

[^1]
## Challenge: Peptide degeneracy

Peptide degeneracy: a single peptide mapped to multiple proteins.


- Shared peptides should belong to all proteins that they can match.


## Challenge: Peptide degeneracy

Peptide degeneracy: a single peptide mapped to multiple proteins.

| Peptides | Proteins | Peptide probabilities <br> ASTSSSSSSSSSNQQTEKETNTPK |  |
| :--- | :--- | :--- | :--- |
| \|P51965|UB2E1_HUMAN |  |  |  |$\quad$| 0.9970 |
| :--- |

Figure: Peptide identifications (Sigma49 data)

- Shared peptides should belong to all proteins that they can match.
- Conditional probability: model the conditional probability of
- one protein being present given a peptide,
- one peptide being present given a protein.


## Existing protein inference algorithms

- ProteinProphet ${ }^{3}$ calculates the conditional probability. Given peptides $i, i=1 \cdots n$, with probabilities $\operatorname{Pr}\left(+\mid S_{i}\right)$ corresponding to a protein, the probability $p$ that this protein is present:

$$
\begin{equation*}
p=1-\prod_{i}^{n}\left[1-\operatorname{Pr}\left(+\mid S_{i}\right)\right] . \tag{1}
\end{equation*}
$$

- Fido ${ }^{4}$ estimates the protein posterior error probability.

$$
\begin{equation*}
p=\operatorname{Pr}(+\mid \text { protein }) . \tag{2}
\end{equation*}
$$

[^2]
## Part II

## Protein linear programming (ProteinLP)

## Model

Table 1. Notations and definitions

| Notations | Definitions |
| :--- | :--- |
| $(1, \ldots, i, \ldots, n)$ | All $n$ peptides identified by peptide identification algorithms |
| $(1, \ldots, j, \ldots, m)$ | All $m$ proteins that might have generated these $n$ peptides |
| $\left(y_{1}, \ldots, y_{i}, \ldots, y_{n}\right)$ | Peptide vector: indicator variables of peptides' presences if peptide $i$ is present, $y_{i}=1 ;$ otherwise $y_{i}=0$ |
| $\left(x_{1}, \ldots, x_{j}, \ldots, x_{m}\right)$ | Protein vector: indicator variables of proteins' presences |
| $\left(z_{1}, \ldots, z_{i}, \ldots, z_{n}\right)$ | The probabilities of peptides' presences estimated by peptide identification algorithms or PeptideProphet |

- $\operatorname{Pr}\left(x_{j}=1\right):$ the probability that protein $j$ is present in the sample.
- $\operatorname{Pr}\left(y_{i}=1, x_{j}=1\right)$ : the probability that peptide $i$ and protein $j$ are present in the sample.

$$
\begin{equation*}
\operatorname{Pr}\left(x_{j}=1\right)=1-\prod_{i=1}^{n}\left[1-\operatorname{Pr}\left(y_{i}=1, x_{j}=1\right)\right] \tag{3}
\end{equation*}
$$

## Model

- From Eq. 3:

$$
\begin{equation*}
\operatorname{Pr}\left(x_{j}=1\right)=1-\prod_{i=1}^{n}\left[1-\operatorname{Pr}\left(y_{i}=1, x_{j}=1\right)\right]=1-\prod_{i=1}^{n} e^{\ln \left[1-\operatorname{Pr}\left(y_{i}=1, x_{j}=1\right)\right]} \tag{4}
\end{equation*}
$$

The protein probability is rewritten as:

$$
\begin{equation*}
\operatorname{Pr}\left(x_{j}=1\right)=1-\prod_{i=1}^{n} e^{p_{i j}} \tag{5}
\end{equation*}
$$

where $p_{i j}:=\ln \left[1-\operatorname{Pr}\left(y_{i}=1, x_{j}=1\right)\right] \leq 0$.

- The peptide probability:

$$
\begin{align*}
\operatorname{Pr}\left(y_{i}=1\right)=1-\prod_{j=1}^{m}\left[1-\operatorname{Pr}\left(y_{i}=1, x_{j}=1\right)\right] & =1-\prod_{j=1}^{m} e^{p_{i j}} .  \tag{6}\\
z_{i} & =1-\prod_{j=1}^{m} e^{p_{i j}} \tag{7}
\end{align*}
$$

## LP formulation

Objective:
Maximize the number of proteins with zero probabilities, while peptide probabilities from joint probabilities should be as close to the input value as possible.

$$
\begin{array}{cl}
\text { Maximize: } & \sum_{j=1}^{m} t_{j}, \\
\text { Subject to: } & \forall i: t_{j} \leq p_{i j} \leq 0, \\
& \forall i: \ln \left(1-z_{i}-\epsilon\right) \leq \sum_{j=1}^{m} p_{i j} \leq \ln \left(1-z_{i}+\epsilon\right), \\
& p_{i j}=0, \quad \text { if protein } j \text { doesn't contain peptide i. } \tag{11}
\end{array}
$$

## LP formulation

- Constraint (11):
- $p_{i j}=0$ if $\operatorname{Pr}\left(y_{i}=1, x_{j}=1\right)=0$.


## LP formulation

- Constraint (11):
$-p_{i j}=0$ if $\operatorname{Pr}\left(y_{i}=1, x_{j}=1\right)=0$.
- Constraint (10) peptide probability:

$$
\begin{align*}
z_{i} \pm \epsilon & =1-\prod_{j=1}^{m} e^{p_{i j}}  \tag{12}\\
\Rightarrow \ln \left(1-z_{i} \pm \epsilon\right) & =\sum_{j=1}^{m} p_{i j} \tag{13}
\end{align*}
$$

## LP formulation solved with GLPK

A standard LP:

$$
\begin{gather*}
\text { Maximize: }  \tag{14}\\
\text { Subject to: } \begin{array}{l}
c^{T} x+c_{0}, \\
\\
A x=b, \\
\\
A x \leq b, \\
L B \leq x \leq U B .
\end{array}  \tag{15}\\
x=\left(p_{11} \cdots p_{1 m} p_{21} \cdots p_{2 m} \cdots p_{n m} t_{1} \cdots t_{m}\right)^{T}  \tag{16}\\
c^{T}=\left(0 \mid \mathbf{1}_{1, m}\right), \quad c_{0}=0  \tag{17}\\
A=\left(\begin{array}{cccc|c}
\mathbf{1}_{1, m} & \mathbf{0} & \cdots & \mathbf{0} \\
\mathbf{0} & \mathbf{1}_{1, m} & \vdots & \mathbf{0} \\
\mathbf{0} & & \mathbf{1}_{1, m} &
\end{array}\right)
\end{gather*}
$$

## LP formulation solved with GLPK

Using results of $P$ from Glpk, joint probability matrix $1-e^{P}$ is computed :

|  | Protein1 | Protein2 | Protein3 | $\cdots$ | Protein $_{m}$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Peptide 1 (0.9) | 0.9 | 0 | 0 | $\ldots$ | 0 |
| Peptide 2 (0.85) | 0.7 | 0.5 | 0 | $\cdots$ | 0 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ |  |
| Peptide n (0.9) | 0 | 0.5 | 0 | $\cdots$ | 0.8 |
| Protein Probabilities | $1-(1-0.9)(1-0.7)$ | 0.75 | 0 | $\cdots$ | 0.8 |

## Peptide degeneracy

- ProteinLP: joint probability $\operatorname{Pr}\left(x_{j}=1, y_{i}=1\right)$. e.g. if peptide $i$ present in more than one protein: $m, n, r$ :

$$
\begin{equation*}
\operatorname{Pr}\left(x_{m}=1, y_{i}=1\right) \cdot \operatorname{Pr}\left(x_{n}=1, y_{i}=1\right) \cdot \operatorname{Pr}\left(x_{r}=1, y_{i}=1\right)>0 \tag{18}
\end{equation*}
$$

- ProteinProphet: taking a weight $w_{i}^{n}$ into account, if peptide $i$ corresponds to N different proteins.

$$
\begin{equation*}
p_{n}=1-\prod_{i}^{n}\left(1-w_{i}^{j} \operatorname{Pr}\left(+\mid S_{i}\right)\right) \quad j=1 \cdots N \tag{19}
\end{equation*}
$$

Combining with Number of Sibling Peptides (NSP): NSP $P_{i}=\sum_{\{m \mid m \neq i\}} p\left(+\mid D_{m}\right)$.

$$
\begin{equation*}
\operatorname{Pr}(+\mid S, N S P)=\frac{\operatorname{Pr}(S \mid+) \operatorname{Pr}(N S P \mid+)}{\operatorname{Pr}(S \mid+) \operatorname{Pr}(N S P \mid+)+\operatorname{Pr}(S \mid-) \operatorname{Pr}(N S P \mid-)} \tag{20}
\end{equation*}
$$

# Part III 

Results

## Datasets

- Ground-truth data: 18 mixtures (Klimek et al., 2008), Sigma49 and yeast (Ramakrishnan et al., 2009a)
- Data without reference sets: DME (Brunner et al., 2007), HumanMD (Ramakrishnan et al., 2009b) and HumanEKC (Ramakrishnan et al., 2009a).


## Sigma49 tested

- Data obtained from http://www.marcottelab.org/MSdata/ .
- Peptide identification: X!Tandem (v2010.10.01.1) (David and Cottrell, 2004).
- GLPK (LPWrapper in OpenMS)
- Proteinlists:
</ProteinIdentification>
<Peptideldentification score_type="XTandem" higher_score_better="true" significance_threshold="0" MZ="667.96337890625" RT="901.678" > <PeptideHit score=" 20.8 " sequence="DQQKDAEGEGLSATTLLPK" charge $=$ " 3 " aa_before="K" aa_after="L" protein_refs="PH_4025" > <UserParam type="float" name="E-Value" value="1.1"/>
</PeptideHit>
</PeptideIdentification>
<PeptideIdentification score_type="XTandem" higher_score_better="true" significance_threshold="0" MZ="408.515991210938" RT="902.077" > <PeptideHit score=" 23.2 " sequence="SPPPSPTTQRR" charge=" 3 " aa_before="R" aa_after="L" protein_refs="PH_432 PH_429 PH_428 PH_4628 PH_5036" >
<UserParam type="float" name="E-Value" value="0.65"/>
</PeptideHit>
</Peptideldentification>


## Validation

With setting a threshold $t$ on the protein probabilities, only positive proteins remain.
False positives can be determined:

- Ground truth datasets.
- Datasets without references - using target-Decoy Analysis.
- Protein database comtaminated with a set of shuffled unreal sequences (decoy database).
- Protein from decoy database is false one.


## Validation

Given a certain probability threshold $\mathrm{t}, F_{t}$ is the number of false positives,

- Fasle Discovery Rate (FDR):

$$
F D R_{t}=\frac{F_{t}}{F_{t}+T_{t}}
$$

- q-values:

$$
q_{t}=\min _{t^{\prime} \leq t} F D R_{t^{\prime}} .
$$

- Posterior error probability (PEP):

$$
P E P=\operatorname{Pr}(+\mid p) .
$$



## Comparison of $q$-values




MSB is MSBayespro ${ }^{5}$.
${ }^{5}$ Yong Fuga Li et al. "A Bayesian approach to protein inference problem in shotgun proteomics". In: Journal of Computational Biology 16.8 (2009), pp. 1183-1193.

## Comparison of $q$-values



## Comparison: the number of degenerate peptides

|  | PP |  | ProteinLP |  | Fido |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | TP | FP | TP | FP | TP | FP |
| Simple proteins | 17 | 8 | 17 | 9 | 17 | 9 |
| Degenerate Proteins | 1 | 5 | 0 | 5 | 1 | 4 |
| Simple proteins | 27 | 1 | 30 | 1 | 30 | 5 |
| Degenerate Proteins | 5 | 10 | 5 | 7 | 5 | 3 |
| Simple proteins | 70 | 0 | 64 | 0 | 111 | 6 |
| Degenerate Proteins | 54 | 0 | 60 | 0 | 7 | 0 |

Table: Accuracy on proteins containing shared peptides with q-value threshold 0.3 for Sigma49 and 0.01 for HumanMD.

## Part IV

## Conclusions

## Conclusions

- Joint probabilities provide the degeneracy information.
- Joint probabilities simplify the optimization problem.
- To do:
- Integrate supplementary information, e.g. protein-protein interaction, by adding linear constraints.
- Considering the parameter $\epsilon$ for different peptide probabilities and protein information.

Thanks for listening.

Questions?


[^0]:    ${ }^{1}$ Ting Huang and Zengyou He. "A linear programming model for protein inference problem in shotgun proteomics." In: Bioinformatics 28.22 (2012), pp. 2956-2962.

[^1]:    ${ }^{2}$ A. Keller et al. "Empirical statistical model to estimate the accuracy of peptide identifications made by MS/MS and database search". In: Analytical chemistry 74.20 (2002), pp. 5383-5392.

[^2]:    ${ }^{3}$ Alexey I Nesvizhskii et al. "A statistical model for identifying proteins by tandem mass spectrometry". In: Analytical chemistry 75.17 (2003), pp. 4646-4658.
    ${ }^{4}$ Oliver Serang, Michael J MacCoss, and William Stafford Noble. "Efficient marginalization to compute protein posterior probabilities from shotgun mass spectrometry data". In: Journal of proteome research 9.10 (2010), pp. 5346-5357.

