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Proteomics

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Exercises 3

1. **Random variables and probability (easy)**

   Let $f(x) = x/15, x = 1, 2, 3, 4, 5$, zero elsewhere, be the probability function of X.

   Find $Pr(X = 1)$ or 2, $Pr(1/2 < X < 5/2)$, and $Pr(1 \leq X \leq 2)$.

2. **Binomial distribution**

   Let X be the number of heads in n=7 independent tosses of an unbiased coin. Find the mean value and the variance of X. What is the probability of $X = 5$?

3. **Poisson distribution**

   In a manuscript, it is discovered that only 13.5% of the pages contain no typing errors. If we assume that the number of errors per page is a random variable with a Poisson distribution, find the percentage of pages that have exactly one error.

4. **Score distribution**

Now the search engine outputs 100 pepetide identifications with scores in descending order. There are 87 identifications scoring larger than s_1, in which 8 false identifications are found. In the following table, the last 13 peptide identifications are listed. What is the q-value of the peptide ID scoring s_8 ?

What is the $FDR[s >= s_8]$? And what is the corresponding FPR?

Peptide identification	Search engine score	True/false
LCEVEEGDKEDVDK	s_1	T
YTAQVDAEEKEDVK	s_2	T
IVADKDYSVTANSK	s_3	T
TGIEIIKK	s_4	T
DLGEEHFK	s_5	T
TASSDTSEELNSQDSPK	s_6	F
GAGGENEPPAAAPEPR	s_7	T
IKDPDAAKPEDWDDR	s_8	T
VDEVGGEALGR	s_9	T
SEEQLKEEGIEYK	s_{10}	F
LHVDPENFK	s_{11}	T
FSTVAGESGSADTVRDPR	s_{12}	T
AEEDEILNR	s_{13}	F

5. **EM algorithm: One step (medium)**

Given $x = [-6, -5, -4, 0, 4, 5, 6]$ and the initial parameters $\mu_1 = -1$ and $\mu_2 = 6$, $\sigma_1 = 2$, $\sigma_2 = 1$, $\pi_1 = \pi_2 = 0.5$.

Perform the first iteration of the EM algorithm, i.e., calculate the responsibilities for the first component [= 1-second] for each datum below, $r_{1,1}$, $r_{1,5}$, $r_{1,6}$, then give MLEs of the new parameters for one component: μ_1, σ_1, π_1.

6. **Working on a discrete joint probability table**

x	y	P(X = x,Y = y)
0	1	0.2
0	2	0.1
1	1	0.0
1	2	0.2
2	1	0.3
2	2	0.2

Given the following setting: The random variable X has a range of $\{0, 1, 2\}$ and the random variable Y has a range of $\{1, 2\}$. The joint distribution of X and Y is given by the above table.

Calculate the following marginal probabilities:

P(X = 0) =

P(X = 1) =

P(X = 2) =

P(Y = 0) =

P(Y = 1) =

P(Y = 2) =

Calculate the conditional probability distribution of X given Y = 2:

$P(X = 0|Y = 2) =$

$P(X = 1|Y = 2) =$

$P(X = 2|Y = 2) =$

Calculate the expectation values:

E(X) =

E(Y) =

E(XY) =

Are X and Y independent?