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Optimization

WS 2014/15

Exercises 4

1. **Branch and Bound**

$$\begin{array}{lccccccccc} \text{max} & 7x_1 & + & 10x_2 & + & 4x_3 & + & 5x_4 & & \\ \text{w.r.t.} & & & & & & & & & \\ & 6x_1 & + & 8x_2 & + & 4x_3 & + & 2x_4 & \leq & 15 \\ x_1, x_2, x_3, x_4 \in \{0, 1\} & & & & & & & & & \end{array}$$

   (a) First install SCIP (http://scip.zib.de/). You can find an example, how to use SCIP, here: http://scip.zib.de/doc/html/SHELL.shtml

   (b) Solve the LP relaxation with SCIP.

   (c) Apply branch and bound to find the optimal solution to the ILP.

   (d) check your solution with the help of SCIP.

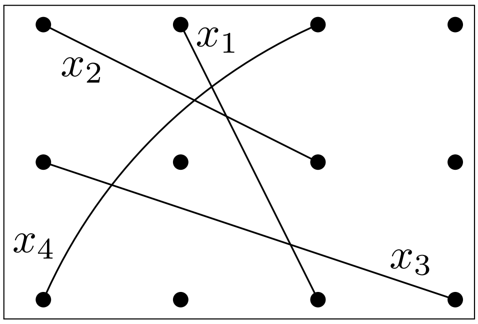
2. **Branch and Cut**
   Apply the cutting plane method to compute an optimal alignment of two sequences "'$ACCA$"' and "'$CACA$"' where a match scores 1 and a mismatch or gap scores 0:

   (a) Draw the alignment graph, the conflict graph, and the pair graph.

   (b) Now start with the trivial (relaxed) LP and add successively clique inequalities which you can find on the longest paths in the pair graph that is labeled with the solution of the last step. Repeat this until you get the optimal alignment.

3. **Branch and Cut**

Given the following alignment graph:



All edges have weight 1.

(a) Try to solve the alignment problem by using branch-and-cut: Add mixed cycle inequalities (see the 'shortest path' method in the script, page 18) to the corresponding (relaxed) LP. Can you reach an optimal solution for the ILP without branching?

(b) Now use branching to solve the problem.

(c) Instead of branching, just add the inequality

$$x_1 + x_2 + x_3 + x_4 \leq 2$$

Can you solve the ILP now?

(d) Prove that the inequality in (c) is facet-defining.

4. **Facets**

Proof the following two lemmas:

Theorem 1. *Let $G = (V, E, H, I)$ be a SEAG with n alignment edges and m interaction matches. Then*

- *$P_{\mathcal{R}}(G)$ is full-dimensional and*
- *the inequality $x_i \leq 1$ is facet-defining iff there is no $e_j \in E$ in conflict with e_i.*

Theorem 2. *Let $G = (V, E, H, I)$ be a SEAG with n alignment edges and m interaction matches. Then*

- *The inequality $x_i \geq 0$ is facet-defining iff e_i is not contained in an interaction match.*
- *For each interaction match $m_{i,j}$ the inequality $x_{i,j} \geq 0$ is facet-defining.*