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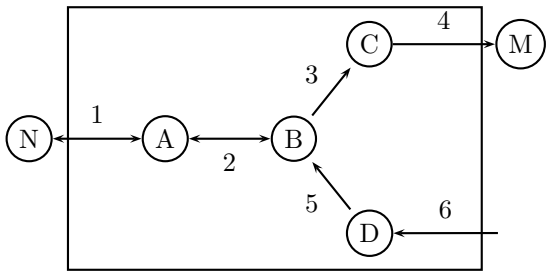
Deadline: January 08, 2015, 11:30 am

Optimization

WS 2014/15

Exercises 2

1. Find the stoichiometric matrix of the following network:
   (Hint: The stoichiometric matrix only has rows corresponding to internal metabolites.)



   (a) Reaction 6 is allowed to carry a flux of at most 1. Maximize the production of metabolite M. Write down the LP-Form and the dual of it.

   (b) Reaction 6 is allowed to carry a flux of at most 1. Maximize the production of metabolite N. Write down the LP-Form and the dual of it.

   Solve the problems of a) and b) with Gurobi (the primal as well as the dual) and analyse your results.

2. Consider the polyhedron $P \subset \mathbb{R}^2$ defined by the system of linear inequalities

$$x_1 + x_2 \leq 4, \quad 0 \leq x_1 \leq 4, \quad 0 \leq x_2 \leq 2 \qquad (*)$$

   (a) Write $(*)$ in the form $Ax \leq b$, for some $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$.

   (b) Determine the bases of $(*)$ and the corresponding basic solutions.

   (c) Which of them are feasible?

   (d) Give for each vertex of P the corresponding feasible bases.

3. Consider the linear program $\max\{c^T v \mid Av \leq b\}$, where

$$A = \begin{pmatrix} -1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad c = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, b = \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \\ 42 \end{pmatrix}.$$

Is $v = \begin{pmatrix} -5 \\ 3 \\ 2 \\ 1 \end{pmatrix}$ a feasible basic solution for the basis $I = \{1, 2, 3, 4\}$? And if so, is it an optimal solution? Prove your results with the help of the theory for the simplex algorithm.

4. Consider the linear optimization problem:

$$\begin{array}{lcl} \max & c_1 x_1 + c_2 x_2 & \\ \text{w.r.t.} & x_1 - x_2 & \leq 1 \\ & x_1, x_2 & \geq 0 \end{array}$$

Determine coefficients (c_1, c_2) of the objective function such that

(a) the problem has a unique optimal solution.

(b) the problem has multiple optimal solutions and the set of optimal solutions is bounded.

(c) the problem has multiple optimal solutions and the set of optimal solutions is unbounded.

(d) the problem has feasible solutions, but no optimal solutions.