

## Optimization

WS 2014/15

### Exercises 0

1.  $A, B, C \in \mathbb{R}^{m \times n}$ . Which of the following statements are always true?  
  $AB = BA$   
  $(AB)C = A(BC)$
2.  $\lambda \in \mathbb{R}$ ,  $v \in \mathbb{R}^n$ ,  $S \in \mathbb{R}^{m \times n}$   
Which of the following statements are always true?  
  $\lambda Sv = S\lambda v$   
  $\lambda Sv = \lambda vS$   
  $\lambda Sv = Sv\lambda$   
  $\lambda Sv = (\lambda S)v$   
  $\lambda Sv = \lambda(Sv)$   
  $\lambda Sv = v\lambda S$
3. Where do the results of the following examples live in (in  $\mathbb{R}$ , in  $\mathbb{R}^n$ , in  $\mathbb{R}^{m \times n}$ , in  $\mathbb{R}^{m' \times n}$  etc.)?
  - $\lambda S$
  - $\lambda v$
  - $Sv$
  - $v^T v$
  - $P \in \mathbb{R}^{m' \times n'}$ :  $SP$
4. Reformulate  $S(\lambda v + w)$
5. If  $A \in \mathbb{R}^{m \times n}$ , where does  $A^T$  live in and if  $A = \begin{pmatrix} a_{11} & \dots & a_{m1} \\ \vdots & & \vdots \\ a_{1n} & \dots & a_{mn} \end{pmatrix}$  how does  $A^T$  look like?
6. What is always true?  
  $(AB)^T = A^T B^T$  or   $(AB)^T = B^T A^T$  or  both  
  $(AB)^{-1} = A^{-1} B^{-1}$  or   $(AB)^{-1} = B^{-1} A^{-1}$  or  both

7. • What is the rank of a matrix?  
 • What does it mean if a matrix is non-singular?

8. Consider the linear optimization problem:

$$\begin{array}{rll} \max & 2x_1 & + & 5x_2 \\ \text{w.r.t.} & x_1 & + & 2x_2 \leq 4 \\ & 4x_1 & + & 3x_2 \leq 12 \\ & & & 2x_2 \leq 3 \\ & x_1, & & x_2 \geq 0 \end{array}$$

- (a) Determine the feasible region.  
 (b) Solve the optimization problem graphically.  
 (c) Solve the problem for the new objective function  $2x_1 + 4x_2$ .  
 (d) What do you notice?

9. Consider now this linear optimization problem:

$$\begin{array}{rll} \max & 2x_1 & + & 5x_2 \\ \text{w.r.t.} & x_1 & - & x_2 \leq -1 \\ & 2x_1 & + & 3x_2 \leq 12 \\ & & & x_2 \leq 2 \\ & & & x_1 \geq 0 \end{array}$$

- (a) Determine the feasible region.  
 (b) Solve the optimization problem graphically.  
 (c) What do you notice?

## Programming exercises

10. Construct in MATLAB the following matrices and vectors:

(a) identity matrix of size  $6 \times 6$

(b)  $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$

(c) The vector  $v = (0, 0, 0, 0, 0, 0, 2)$

(d) The vector  $v = (1, 1, 1, 1, 1, 2)$

*Hint:* use the commands `eye`, `zeros` and `ones`.

11. Check if  $A$  is invertible. If the inverse exists, compute it.

12. Solve the following linear equations:

(a)  $\begin{pmatrix} 1 & 2 & 0 \\ 4 & 0 & 6 \\ 0 & 8 & 9 \end{pmatrix} x = \begin{pmatrix} 2 \\ -10 \\ 33 \end{pmatrix}$

(b)  $\begin{pmatrix} 16 & 2 & 3 & 0 \\ 5 & 11 & 10 & 8 \\ 9 & 7 & 6 & 12 \\ 4 & 14 & 15 & 1 \end{pmatrix} x = \begin{pmatrix} 29 \\ 89 \\ 89 \\ 81 \end{pmatrix}$

(c)  $\begin{pmatrix} 16 & 5 & 9 & 4 \\ 2 & 11 & 7 & 14 \\ 3 & 10 & 6 & 15 \\ 0 & 8 & 12 & 1 \end{pmatrix} x = \begin{pmatrix} 69 \\ 101 \\ 101 \\ 56 \end{pmatrix}$

13. Install gurobi

(a) go to the webpage <http://www.gurobi.com/> and download the gurobi solver

(b) install it on your computer

(c) try to run the example (`coins.lp`)

Under "Quick Start Guide" you will find some useful tips.