FREIE UNIVERSITÄT BERLIN
Fachbereich Mathematik und Informatik
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Optimization

WS 2012/13

1st exam

last name, first name	Matr.-Nr.

Result:

	points	max.
1		23
2		15
3		15
4		12
5		15
6		10
Σ		90

You have 90 minutes for the exam. Please write Matrikelnummer and name on each sheet you hand in.

1. [*13+10=23 Points*] **(Linear Programming and Duality)**

   A plant makes aluminum and copper wire. Each kg of aluminum wire requires 5 kWh of electricity and 1/4 hr. of labor. Each kg of copper wire requires 2 kWh of electricity and 1/2 hr. of labor. Production of copper wire is restricted by the fact that raw materials are available to produce at most 60 kg/day. Electricity is limited to 500 kWh/day and labor to 40 hrs./day. If the profit from aluminum wire is $0.25/kg. and the profit from copper is $0.40/kg., how much of each should be produced to maximize profit and what is the maximum profit?

   (a)
   - Model the problem as a linear program.
   - Solve the linear program graphically to compute the coordinates of the optimal solution as well as its value.

   (b)
   - Formulate the dual of your LP.
   - State the *strong duality* theorem and *weak duality* theorem in linear programming.

2. [*10+5=15 Points*] **(Combinatorial Optimization: Modeling)**

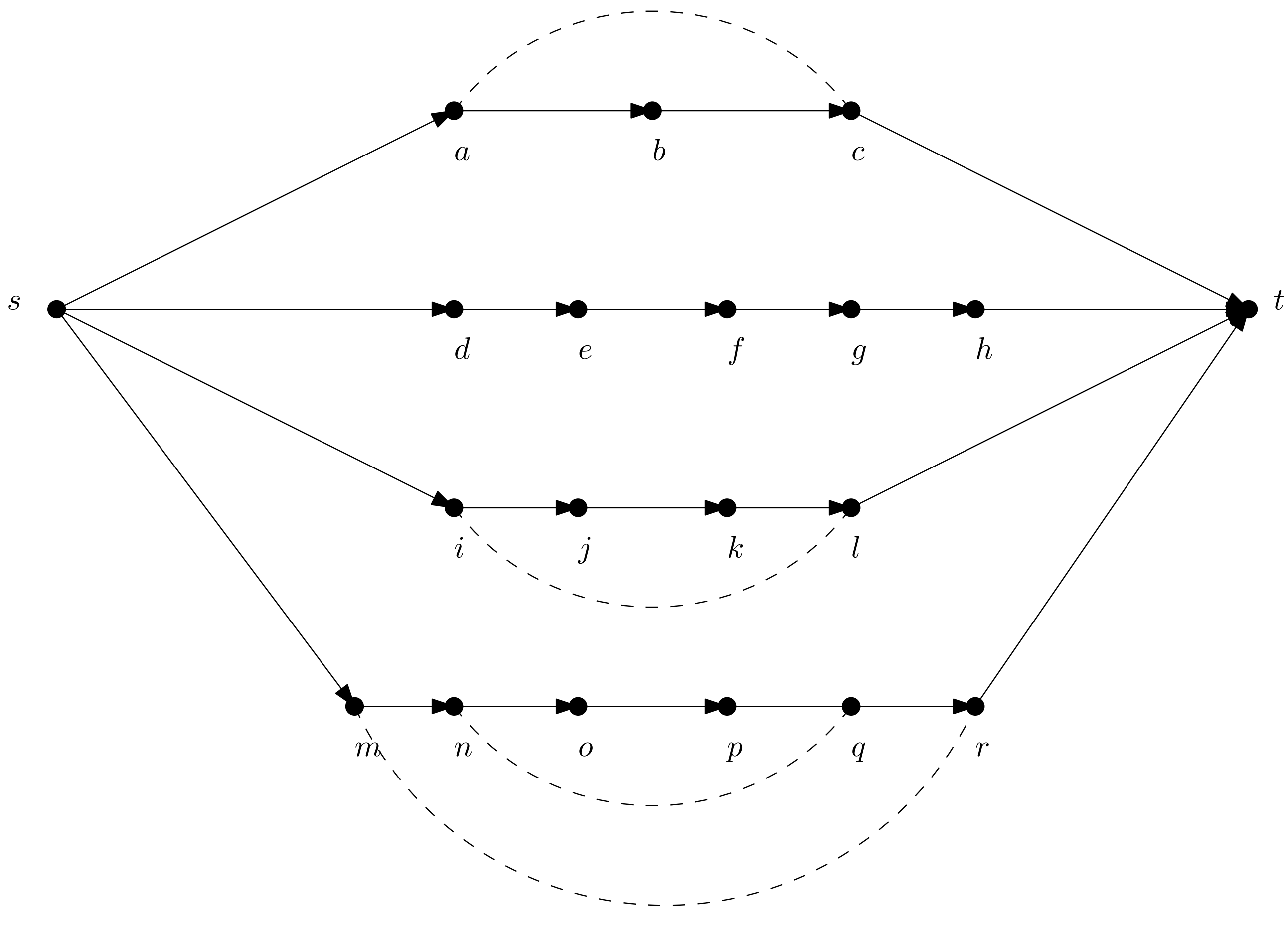
Given a graph $G = (V, E)$, two vertices $s, t \in V$, and a set of pairs of vertices $C \subset V \times V$, the *shortest antisymmetric path problem* consists in finding a path from s to t with the minimal number of edges, which contains at most one vertex from each pair of vertices in C.

(a) Give an integer linear programming formulation for the antisymmetric shortest path problem.

(b) How can you, in general prove, whether an inequality for a combinatorial optimization problem is facet-defining?

3. [*15 Points*] **(Combinatorial Optimization: Branch-and-cut)**

Solve the *shortest antisymmetric path problem* for the small instance given below by branch-and-cut where you use the forbidden pairs inequalities as cutting planes. The forbidden pairs of vertices are denoted by dashed lines.



4. [*6+6=12 Points*] **(Combinatorial Optimization: Lagrange relaxation)**

Assume you are given the optimization problem

$$\min c^T x \tag{1}$$
$$Ax \geq b \tag{2}$$
$$Dx \geq d \tag{3}$$
$$x \text{ integer} \tag{4}$$

with A, D, b, c, d having integer entries. Let Z_{IP} be the optimal value to the ILP above and let

$$X = \{x \text{ integral } \mid Dx \geq d\}.$$

We assume that optimizing over the set X can be done very easily, whereas adding the bad constraints $Ax \geq b$ makes the problem intractable.

(a) Formulate the *Lagrangian Dual* of the above problem.

(b) Show that the solution of the Lagrangian dual, Z_D, is a lower bound for Z_{IP}

5. [*15 Points*] **(Constraint programming)**

Consider the constraint satisfaction problem

$$\begin{gathered} x_1 \in \{0,1,2\},\ \ x_2 \in \{1,2,3\},\ \ x_3 \in \{0,1,2,3\} \\ C_1 : x_1 \geq 1,\ \ C_2 : x_2 < 3, \\ C_{1,3} : x_1 = x_3, \quad C_{2,3} : x_2 < x_3. \end{gathered} \tag{CSP1}$$

(a) Draw the corresponding constraint graph.

(b) Make the graph node consistent.

(c) Consider the arcs one by one and make the graph arc consistent.

Consider now the problem

$$\begin{gathered} x_1 \in \{1,2\},\ \ x_2 \in \{1,2\},\ \ x_3 \in \{1,2\}, \\ C_{1,2} : x_1 \neq x_2,\ \ C_{1,3} : x_1 = x_3,\ \ C_{2,3} : x_2 \leq x_3. \end{gathered} \tag{CSP2}$$

(d) Is the corresponding constraint graph arc consistent? Justify your answer.

(e) Apply the forward checking algorithm to find all solutions.

6. [*10 Points*] **(Metaheuristics)**

(a) What is the difference between complete and approximate algorithms for discrete optimization problems?

(b) Briefly describe two metaheuristics of your choice.

(Supplementary sheet 1)

(Supplementary sheet 2)