FREIE UNIVERSITÄT BERLIN
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Optimization

WS 2012/13

2nd exam

last name, first name	Matr.-Nr.

Result:

	points	max.
1		20
2		15
3		15
4		18
5		16
6		12
Σ		96

You have 90 minutes for the exam. Please write Matrikelnummer and name on each sheet you hand in.

1. [*20 Points*] **(Linear Programming)**

A plant produces two types of cell phones, A and B. There are two production lines, one dedicated to producing cell phones of type A, the other to producing cell phones of type B. The capacity of the production line for A is 80 units per day, the capacity of the production line for B is 100 units per day. Type A requires 40 minutes of labor whereas type B requires 80 minutes of labor. Presently, there is a maximum of 80 hours of labor per day. According to market analysts at least 20% of the produced cell phones have to be of type B. Profit contributions are \$20 per cell phones of type A produced and \$25 per type B produced. What should the daily production be?

   (a) Formulate the problem as a linear program.

   (b) Solve the linear program graphically to compute the coordinates of the optimal solution as well as its value.

   (c) Give all basic feasible solutions and the corresponding bases.

   (d) Modify the profit contributions (both must remain strictly positive) such that the number of optimal solutions for your modified LP becomes infinite.

   (e) Write the linear program in the form: $\min\{c^T x \mid Ax = b, x \geq 0\}$.

2. [*15 Points*] **(Combinatorial Optimization: Modeling)**

   Assume you are given p sequences, which are assumed without loss of generality to each have length N, and a motif length $l < N$. The goal is to find a subsequence s_i of length l in *each* sequence i so as to minimize the sum of the pairwise Hamming distances between the subsequences.

   Hint: For the given problem you can define a p-partite graph, where the nodes correspond to starting positions of the motif and the weighted edges correspond to the Hamming distance between the motifs starting at the respective positions.

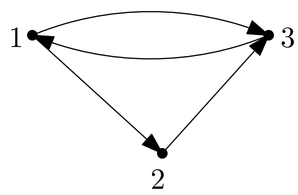
   (a) Formulate an ILP which solves the motif finding problem and explain the meaning of your variables and objective function.

3. [*5+10=15 Points*] **(Combinatorial Optimization: Branch-and-cut)**

The Acyclic Subdigraph Problem (ASP) is defined as follows: Given a directed graph (digraph) $D = (V, E)$ and a edge weight function $c : E \to \mathbb{Z}$, find a acyclic subgraph having maximum weight. The ASP can be formulated as an ILP using the class of dicycle inequalities:

$$\sum_{(i,j)\in C} x_{ij} \leq |C| - 1$$

for each directed cycle C.



(a) For the given digraph (weights are all the same), formulate an ILP for the ASP.

(b) Are the dicycle inequalities facet-defining in the example? If no, give an argument why not. If yes, prove it.

4. [*5+8+5=18 Points*] **(Combinatorial Optimization: Lagrange relaxation)**

Consider the following problem:

$$\begin{array}{rrcrcr} \min & -3x_1 & + & 2x_2 & & \\ \text{w.r.t.} & -4x_1 & + & 3x_2 & \geq & -6 \\ & x_1 & - & x_2 & \leq & 2 \\ & 2x_1 & + & 6x_2 & \geq & 3 \\ & x_1 & + & 6x_2 & \leq & 15 \\ & & & x_1, x_2 & \geq & 0 \\ & & & x_1, x_2 & \in & \mathbb{Z} \end{array}$$

Apply Lagrangian relaxation by relaxing the first inequality:

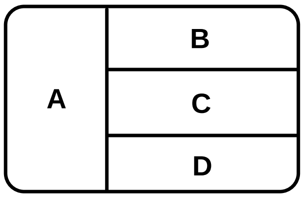
(a) State the Lagrange problem and the Lagrangian Dual.

(b) Show the polytopes of the ILP, the LP relaxation and the Lagrange problem.

(c) State the relationship between Z_{IP} and Z_D for a minimization problem and prove it (Z_{IP} and Z_D denote the optimum values of the ILP and Lagrangian dual).

5. [*16 Points*] **(Constraint programming)**

Suppose we want to color the map



using at most 3 colors.

(a) Model the problem as a finite domain constraint satisfaction problem (do not use 0-1 variables).

(b) Solve the model step by step using

i. naive backtracking

ii. forward checking

(c) Model the problem with a 0-1 linear program that allows to find the minimal number of colors that is necessary to color the map.

(d) What does it mean that a constraint satisfaction problem is

i. node consistent ?

ii. arc consistent ?

6. [*12 Points*] **(Metaheuristics)**

(a) What is a metaheuristic for a discrete optimisation problem (up to 4 characteristics)?

(b) Explain the terms "intensification" and "diversification" (in the context of metaheuristics).

(c) Briefly describe the metaheuristic "Evolutionary computation" (up to 4 characteristics).

(Supplementary sheet 1)

(Supplementary sheet 2)