

Optimization

WS 2013/14

Exercises 1

1. Transform the linear optimization problem

$$\begin{array}{rcl} \min & 3x_1 & + \quad x_2 \\ \text{w.r.t.} & 2x_1 & + \quad 5x_2 \leq 12 \\ & 3x_1 & + \quad x_2 = 9 \\ & x_1 & \geq 0 \end{array}$$

to the canonical form $\max\{c^T x \mid Ax = b, x \geq 0\}$.

2. Consider the linear optimization problem:

$$\begin{array}{rcl} \max & 2x_1 & + \quad 5x_2 \\ \text{w.r.t.} & x_1 & + \quad 2x_2 \leq 4 \\ & 4x_1 & + \quad 3x_2 \leq 12 \\ & & 2x_2 \leq 3 \\ & x_1, & x_2 \geq 0 \end{array}$$

- (a) Determine the feasible region.
- (b) Solve the optimization problem graphically.
- (c) Solve the problem for the new objective function $x_1 + 2x_2$.
- (d) What do you notice?

3. Profit optimization

A company produces cat food and dog food. There are three different types of machines which are needed. The first one cut the meat, the second one is only for the cat food to get more flavour in the food, because cats are more delicate than dogs. The third one packages the food in cans.

Cat food need for one unit only 20 minutes on the first machine but 10 minutes extra on the second. To package the food takes 30 minutes on the last machine.

To produce dog food there are 30 minutes needed on the first machine and 80 minutes on the last one.

One unit of cat food can be sold for 6 Euro, one unit of dog food for 2 Euro.

Because the factory don't want the cats and dogs to be hungry they produce at least one unit of cat food and one of dog food.

Since there are also other kind of foods which are produced in the factory there is a time-limit on how much the machines can be used:

- machine 1 can be used 2 hours
- machine 2 can be used 3.5 hours
- machine 3 can be used 4 hours

How much cat food and how much dog food should they produce if they want to maximize their profit?

- (a) Formulate the problem as a linear program.
- (b) Solve the linear program graphically to compute the coordinates of the optimal solution as well as its value.