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Optimization

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Exercises 5

1. **Bin Packing**

   Consider the following variant of the *bin packing* problem:

   - Pack n items of size $g_i, i = 1, \dots, n$, into (at most) n bins, each of capacity c.
   - Put the first m items into different bins.
   - Find the minimal number of bins necessary.

   Model the problem in integer linear programming

2. **IP Formulations**
   Suppose that you are interested in choosing a set of investments $\{1, \dots, 7\}$ using $0-1$ variables. Model the following constraints:

   (a) You cannot invest in all of them.
   (b) You must choose at least one of them.
   (c) Investment 1 cannot be chosen if investment 3 is chosen.
   (d) Investment 4 can be chosen only if investment 2 is also chosen.
   (e) You must choose either both investments 1 and 5 or neither.
   (f) You must choose either at least one of the investments $1, 2, 3$ or at least two investments from $2, 4, 5, 6$.

3. ***n*-Queens Problem**
   Model the n-queens problem (as an integer linear program):
   Place n queens on an $n \times n$ chess board such that in each line (horizontal, vertical and diagonal) only one queen is allowed.

4. **SCIP**

Use SCIP to solve the following exercise:

There are 3 depots and 4 customers and each customer ordered 1 package.
f_i denotes the costs to open the depot i, c_{ij} are the costs for delivering the package from depot i to the customer j.
Each customer has to get his package and the aim is to minimize the costs. The given values are: $f_1 = 3,\ f_2 = 2,\ f_3 = 4$ and

$c_{11} = 2$	$c_{21} = 3$	$c_{31} = 1.5$
$c_{12} = 2.5$	$c_{22} = 4.5$	$c_{32} = 1$
$c_{13} = 2$	$c_{23} = 4.5$	$c_{33} = 1.5$
$c_{14} = 3$	$c_{24} = 5$	$c_{34} = 2$

Of course customer j can only get his package from depot i if this depot is open. Thus:
$x_{ij} \leq y_i$, where

$$y_i = \begin{cases} 1, & \text{depot } i \text{ is open} \\ 0, & \text{else} \end{cases}$$

Don't use the command "Binary" but Bounds (≥ 0 and ≤ 1) for the variables x_{ij} and y_i. Formulate the problem in two different ways and compare the results:

(a) $x_{ij} \leq y_i$ for every j and i.

(b) rewrite the above formulation such that $\sum_{j=1}^{4} x_{ij} \leq 4y_i$ for all $i = \{1, 2, 3\}$.