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Optimization

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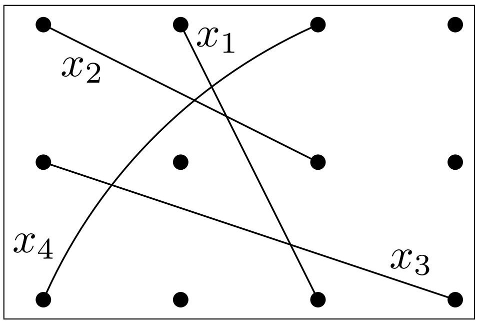
Exercises 4

1. **Branch and Cut (NIVEAU I)**
   Given the following alignment graph:



All edges have weight 1.

(a) Try to solve the alignment problem by using branch-and-cut: Add mixed cycle inequalities to the corresponding (relaxed) LP. Can you reach an optimal solution for the ILP without branching?

(b) Now use branching to solve the problem.

(c) Instead of branching, just add the inequality

$$x_1 + x_2 + x_3 + x_4 \leq 2$$

Can you solve the ILP now?

(d) Prove that the inequality in (c) is facet-defining.

2. **Lagrangean Relaxation I (NIVEAU I)**
Consider the following problem

$$\begin{array}{lrcrcr} \min & 2x_1 & - & 3x_2 & & \\ \text{w.r.t.} & 3x_1 & - & 4x_2 & \geq & -6 \\ & -x_1 & + & x_2 & \leq & 2 \\ & 6x_1 & + & 2x_2 & \geq & 3 \\ & 6x_1 & + & x_2 & \leq & 15 \\ & & & x_1, x_2 & \geq & 0 \\ & & & x_1, x_2 & \in & \mathbb{Z} \end{array}$$

(a) Draw the corresponding polytope and determine graphically the optimal solution Z_{IP} of the original problem and Z_{LP}, the solution of the LP-relaxation.

(b) Now apply lagrangean relaxation by relaxing the first inequality. Draw the polytope of the relaxed ILP. Determine the set X of feasible solutions for the relaxed problem.

(c) The new objective function is then:

$$Z(P) = \min_{(x_1,x_2)\in X} 2x_1 - 3x_2 + p(-6 - 3x_1 + 4x_2)$$

Calculate $Z_D = \max_{p\geq 0} Z(p)$ and compare this value to Z_{IP} and Z_{LP}. (To obtain Z_D, draw the graphs of the function $f(p) = 2x_1 - 3x_2 + p(-6 - 3x_1 + 4x_2)$ for all $(x_1, x_2) \in X$.)

(d) repeat a-c for the objective functions $-x_1 + x_2$ and $-x_1 - x_2$ and compare Z_{LP}, Z_D, and Z_{IP}.

3. **Lagrangean Relaxation II (NIVEAU I)**
   Prove Lemma 1 (see script page 4001) stating that (in case of a minimization problem) if $\lambda \geq 0$, then $Z(\lambda) \leq Z_{IP}$, where Z_{IP} is the optimal value of an original ILP and $Z(\lambda)$ is the optimal value of the relaxed problem for a given value of the Lagrangean multiplier λ.

4. **Facets (NIVEAU I)**
   Proof the following two lemmas:

   Lemma 1 *Let $G = (V, E, H, I)$ be a SEAG with n alignment edges and m interaction matches. Then*

   - *$P_{\mathcal{R}}(G)$ is full-dimensional and*
   - *the inequality $x_i \leq 1$ is facet-defining iff there is no $e_j \in E$ in conflict with e_i.*

   Lemma 2 *Let $G = (V, E, H, I)$ be a SEAG with n alignment edges and m interaction matches. Then*

   - *The inequality $x_i \geq 0$ is facet-defining iff e_i is not contained in an interaction match.*
   - *For each interaction match $m_{i,j}$ the inequality $x_{i,j} \geq 0$ is facet-defining.*