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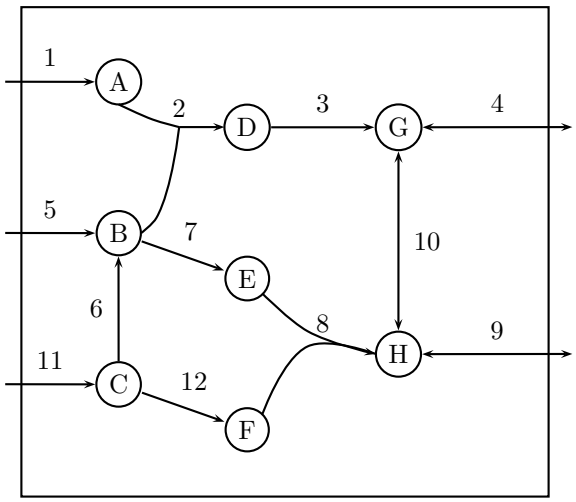
December 18, 2013

Optimization

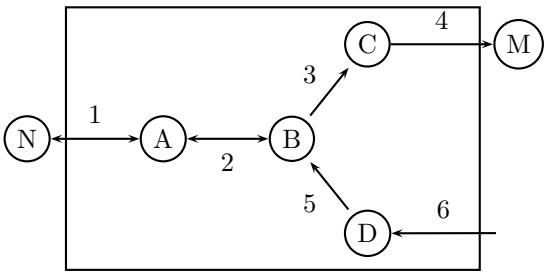
WS 2013/14

Exercises 2

1. Find the stoichiometric matrix of this network:



2. Find the stoichiometric matrix of the following network:
   (Hint: The stoichiometric matrix only has rows corresponding to internal metabolites.)



(a) Reaction 6 is allowed to carry a flux of at most 1. Maximize the production of metabolite M. Write down the LP-Form and the dual of it.

(b) Reaction 6 is allowed to carry a flux of at most 1. Maximize the production of metabolite N. Write down the LP-Form and the dual of it.

Solve the problems of a) and b) with Gurobi (the primal as well as the dual) and analyse your results.

3. Niveau II
Let $S \in \mathbb{R}^{m \times n}$ be a stoichiometric matrix. If $v \in \mathbb{R}^n$ full fills $Sv = 0$, with $v_{\text{Irr}} \geq 0$, then v is a feasible flux vector. v_{Irr} denotes the set of irreversible reactions.
Proof or disprove the following statement:

$$\begin{aligned} & V = \{v_{\text{s}} \mid Sv = 0,\ v_{\text{Irr}} \geq 0, v_{\text{r}} = 0,\ v_s \geq 0\} \text{ is unbounded} \\ \Leftrightarrow\ & P = \{v \in \mathbb{R}^n \mid Sv = 0,\ v_{\text{Irr}} \geq 0, v_{\text{r}} = 0, v_{\text{s}} \geq 1\} \neq \emptyset \end{aligned}$$

4. Consider the linear program $\max\{c^T v \mid Av \leq b\}$, where

$$A = \begin{pmatrix} -1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad c = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, b = \begin{pmatrix} 4 \\ 3 \\ 2 \\ 1 \\ 42 \end{pmatrix}.$$

Is $v = \begin{pmatrix} -5 \\ 3 \\ 2 \\ 1 \end{pmatrix}$ a feasible basic solution for the basis $I = \{1, 2, 3, 4\}$? And if so, is it an optimal solution? Proof your results with the help of the theory for the simplex algorithm.