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January 28, 2013

Optimization

WS 2012/13

Exercises 5

1. **Lagrangean Relaxation I (NIVEAU I)**
   Consider the following problem

$$
\begin{array}{rrcrcr}
\min & 2x_1 & - & 3x_2 & & \\
\text{w.r.t.} & 3x_1 & - & 4x_2 & \geq & -6 \\
 & -x_1 & + & x_2 & \leq & 2 \\
 & 6x_1 & + & 2x_2 & \geq & 3 \\
 & 6x_1 & + & x_2 & \leq & 15 \\
 & & & x_1, x_2 & \geq & 0 \\
 & & & x_1, x_2 & \in & \mathbb{Z}
\end{array}
$$

   (a) Draw the corresponding polytope and determine graphically the optimal solution Z_{IP} of the original problem and Z_{LP}, the solution of the LP-relaxation.

   (b) Now apply lagrangean relaxation by relaxing the first inequality. Draw the polytope of the relaxed ILP. Determine the set X of feasible solutions for the relaxed problem.

   (c) The new objective function is then:

$$Z(P) = \min_{(x_1,x_2)\in X} 2x_1 - 3x_2 + p(-6 - 3x_1 + 4x_2)$$

   .

   Calculate $Z_D = \max_{p\geq 0} Z(p)$ and compare this value to Z_{IP} and Z_{LP}. (To obtain Z_D, draw the graphs of the function $f(p) = 2x_1 - 3x_2 + p(-6 - 3x_1 + 4x_2)$ for all $(x_1, x_2) \in X$.)

   (d) repeat a-c for the objective functions $-x_1 + x_2$ and $-x_1 - x_2$ and compare Z_{LP}, Z_D, and Z_{IP}.

2. **Lagrangean Relaxation II (NIVEAU I)**
   Prove Lemma 1 (see script page 4001) stating that (in case of a minimization problem) if $\lambda \geq 0$, then $Z(\lambda) \leq Z_{IP}$, where Z_{IP} is the optimal value of an original ILP and $Z(\lambda)$ is the optimal value of the relaxed problem for a given value of the Lagrangean multiplier λ.

3. **Inverse Queens Problem (NIVEAU I)**

   The *inverse queens problem* consists in placing n queens on a $n \times n$ chess board, one queen per row, such that each pair is either in the same column or in the same diagonal.

   (a) Model the problem as a constraint satisfaction problem.

   (b) Solve the problem for $n = 4$ by

   - forward checking
   - partial lookahead

   assuming that the first queen is placed in column 2.

4. **Task Scheduling (NIVEAU I)**

   Suppose we have a set of activities, each with a specified duration. There are precedence constraints between the activities, such that if task A precedes task B, then task B cannot start before task A ends.

Task	Duration	Precedes
A	3	B,C
B	2	D
C	4	D
D	2	

   (a) Model the problem as a constraint satisfaction problem.

   (b) Add two artificial tasks *Start* and *End* to model the beginning and the end of the project.

   (c) Apply arc consistency to reduce the domains of the variables.

   (d) What further reduction can be obtained by fixing the end of the project to the minimum possible value?