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Optimization

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Exercises 4

1. **Critical Mixed Cycles (NIVEAU II)**
   Prove the following lemma (see lecture script):

   A subset $T \subseteq E$ is a trace, if and only if $G' = (V, T, H)$ does not contain a critical mixed cycle.

2. **Branch and Cut (NIVEAU I)**
   Apply the cutting plane method to compute an optimal alignment of two sequences "‘$ACCA$"’ and "‘$CACA$"’ where a match scores 1 and a mismatch or gap scores 0:

   (a) Draw the alignment graph, the conflict graph, and the pair graph.

   (b) Now start with the trivial (relaxed) LP and add successively clique inequalities which you can find on the longest paths in the pair graph that is labeled with the solution of the last step. Repeat this until you get the optimal alignment.

3. **Facets (NIVEAU I)**

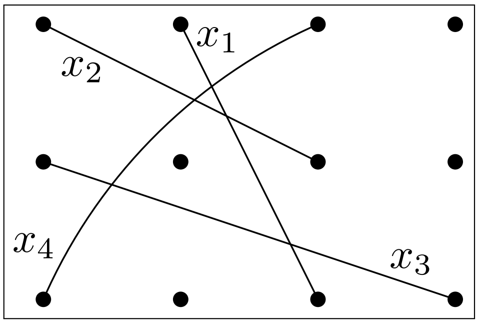
   Lemma 1. *Let $G = (V, E, H, I)$ be a SEAG with n alignment edges and m interaction matches. Then*

   - *$P_{\mathcal{R}}(G)$ is full-dimensional and*
   - *the inequality $x_i \leq 1$ is facet-defining iff there is no $e_j \in E$ in conflict with e_i.*

   Lemma 2. *Let $G = (V, E, H, I)$ be a SEAG with n alignment edges and m interaction matches. Then*

   - *The inequality $x_i \geq 0$ is facet-defining iff e_i is not contained in an interaction match.*
   - *For each interaction match $m_{i,j}$ the inequality $x_{i,j} \geq 0$ is facet-defining.*

4. **Branch and Cut (NIVEAU I)**
Given the following alignment graph:



All edges have weight 1.

(a) Try to solve the alignment problem by using branch-and-cut: Add mixed cycle inequalities (see the 'shortest path' method in the script, page 18) to the corresponding (relaxed) LP. Can you reach an optimal solution for the ILP without branching?

(b) Now use branching to solve the problem.

(c) Instead of branching, just add the inequality

$$x_1 + x_2 + x_3 + x_4 \leq 2$$

Can you solve the ILP now?

(d) Prove that the inequality in (c) is facet-defining.