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Optimization

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Exercises 1

1. Transform the linear optimization problem

$$\begin{array}{rrcrcl} \min & 2x_1 & + & 3x_2 & & \\ \text{w.r.t.} & 3x_1 & + & 6x_2 & \leq & 7 \\ & 2x_1 & + & 2x_2 & = & 5 \\ & x_2 & & & \geq & 0 \end{array}$$

   to the canonical form $\max\{c^T x \mid Ax = b, x \geq 0\}$.

2. Consider the linear optimization problem:

$$\begin{array}{rrcrcl} \max & 3x_1 & + & 4x_2 & & \\ \text{w.r.t.} & 3x_1 & + & 2x_2 & \leq & 12 \\ & 5x_1 & + & 10x_2 & \leq & 30 \\ & & & 2x_2 & \leq & 5 \\ & x_1, & & x_2 & \geq & 0 \end{array}$$

   (a) Determine the feasible region.

   (b) Solve the optimization problem graphically.

   (c) Solve the problem for the new objective function $6x_1 + 12x_2$.

3. **Profit optimization**
   A plant produces two types of refrigerators, A and B. There are two production lines, one dedicated to producing refrigerators of Type A, the other to producing refrigerators of type B. The capacity of the production line for A is 60 units per day, the capacity of the production line for B is 50 units per day. Type A requires 20 minutes of labor whereas type B requires 40 minutes of labor. Presently, there is a maximum of 40 hours of labor per day. According to national environment protection laws at least 50% of the produced refigerators has to be of type B. Profit contributions are \$20 per refrigerator of type A produced and \$25 per type B produced. What should the daily production be?

   (a) Formulate the problem as a linear program.

   (b) Solve the linear program graphically to compute the coordinates of the optimal solution as well as its value.