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Discrete Mathematics for Bioinformatics (P1)

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Exercises 8

1. **Bases and Basic Solutions (Niveau I)**
   Consider the polyhedron $P \subset \mathbb{R}^2$ defined by the system of linear inequalities

   $$x_1 + x_2 \leq 4, \;\; 0 \leq x_1 \leq 4, \;\; 0 \leq x_2 \leq 2 \qquad (*)$$

   (a) Write (*) in the form $Ax \leq b$, for some $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$.

   (b) Determine the bases of (*) and the corresponding basic solutions.

   (c) Which of them are feasible?

   (d) Give for each vertex of P the corresponding feasible bases.

2. **Simplex Algorithm (Niveau I)**
   As the year draws to a close you plan a cool New Year's Eve party where you invite some of your best friends. But you are not a really good friend that pays for the drinks, you want to sell them at your party. You will sell coke and beer and you want to make money. You will make 1 Euro profit per liter coke and 2 Euro per liter beer. In your car you can transport 150kg of drinks. While coke comes in light plastic bottles it weighs 1kg per liter whereas the heavy glass bottles result in a weight of 1.5 kg per liter beer. The store had only 35 liters of your favorite beer and you had to promise your mother that at least 2/3 of the drinks will be non-alcoholic. How many liters of coke and beer should you buy to maximize your profit?

   (a) Model the problem as a linear optimization problem.

   (b) Solve it by applying the Simplex method as discussed in the lecture (You may skip phase I and start without any drinks).

3. **Duality (Niveau II)**
A farmer owns 12 acres of land. He can cultivate potatoes, corn and wheat. Given the data

	Potatoes	Corn	Wheat	Available
Cultivation costs (in €/a.)	24	36	18	360 €
Labor hours per a.	6	6	2	48 hours
Profit (in €/a.)	30	40	20	

what should he plant in order to maximize the profit ?

(a) Model the problem as a linear optimization problem.

(b) Formulate the dual problem and give an economic interpretation.

(c) Use lp_solve to solve both problems and compare their optimal values.