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Discrete Mathematics for Bioinformatics (P1)

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Exercises 7

1. **Convexity of LP (Niveau I)**
   A set S is convex if x_1 and $x_2 \in S$ implies that $x = \lambda x_1 + (1-\lambda)x_2 \in S$ for $0 \leq \lambda \leq 1$.

   Prove the following two Theorems:

   (a) A hyperplane is a convex set.

   (b) Consider the LP:

   $$\begin{array}{rrcl} \max & c^T x & & \\ \text{w.r.t.} & Ax & \leq & b \\ & x & \geq & 0 \end{array}$$

   Let x_1 and x_2 be two feasible solutions. Show that, if $c^T x_1 = k$ and $c^T x_2 = k$ then $c^T x_i = k$ for every point x_i on the line joining x_1 and x_2.

2. **Bases and Basic Solutions (Niveau I)**
   Consider the polyhedron $P \subset \mathbb{R}^2$ defined by the system of linear inequalities

   $$x_1 + x_2 \leq 4, \;\; 0 \leq x_1 \leq 4, \;\; 0 \leq x_2 \leq 2 \qquad (*)$$

   (a) Write (*) in the form $Ax \leq b$, for some $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$.

   (b) Determine the bases of (*) and the corresponding basic solutions.

   (c) Which of them are feasible?

   (d) Give for each vertex of P the corresponding feasible bases.

3. **Simplex Algorithm (Niveau I)**
A farmer owns 100 acres of land. He can cultivate potatoes or corn.
Given the data

	Potatoes	Corn	Available
Cultivation costs (in k€/a.)	1	2	110 k€
Working days per a.	1	4	160 working days
Profit (in k€/a.)	1	3	

what should he plant in order to maximize his income ?

(a) Model the problem as a linear optimization problem.

(b) Solve it by applying the Simplex method (You may skip phase I and start with solution 0).