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Discrete Mathematics for Bioinformatics (P1)

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Exercises 6

1. Transform the linear optimization problem

$$\begin{array}{rrcrcl} \min & 2x_1 & + & 3x_2 & & \\ \text{w.r.t.} & 3x_1 & + & 6x_2 & \leq & 7 \\ & 2x_1 & + & 2x_2 & = & 5 \\ & x_2 & & & \geq & 0 \end{array}$$

   to the canonical form $\max\{c^T x \mid Ax = b, x \geq 0\}$.

2. Consider the linear optimization problem:

$$\begin{array}{rrcrcl} \max & 3x_1 & + & 4x_2 & & \\ \text{w.r.t.} & 3x_1 & + & 2x_2 & \leq & 12 \\ & 5x_1 & + & 10x_2 & \leq & 30 \\ & & & 2x_2 & \leq & 5 \\ & x_1, & & x_2 & \geq & 0 \end{array}$$

   (a) Determine the feasible region.
   (b) Solve the optimization problem graphically.
   (c) Solve the problem for the new objective function $6x_1 + 12x_2$.

3. Consider the linear optimization problem:

$$\begin{array}{rrcrcl} \max & c_1x_1 & + & c_2x_2 & & \\ \text{w.r.t.} & x_1 & - & x_2 & \leq & 1 \\ & x_1, & & x_2 & \geq & 0 \end{array}$$

   Determine coefficients (c_1, c_2) of the objective function such that

   (a) the problem has a unique optimal solution.
   (b) the problem has multiple optimal solutions and the set of optimal solutions is bounded.
   (c) the problem has multiple optimal solutions and the set of optimal solutions is unbounded.
   (d) the problem has feasible solutions, but no optimal solutions.

Finally, add one constraint so that the problem becomes infeasible.

4. **Profit optimization**

   A plant produces two types of refrigerators, A and B. There are two production lines, one dedicated to producing refrigerators of Type A, the other to producing refrigerators of type B. The capacity of the production line for A is 60 units per day, the capacity of the production line for B is 50 units per day. Type A requires 20 minutes of labor whereas type B requires 40 minutes of labor. Presently, there is a maximum of 40 hours of labor per day. According to national environment protection laws at least 50% of the produced refigerators has to be of type B. Profit contributions are \$20 per refrigerator of type A produced and \$25 per type B produced. What should the daily production be?

   (a) Formulate the problem as a linear program.

   (b) Solve the linear program graphically to compute the coordinates of the optimal solution as well as its value.