

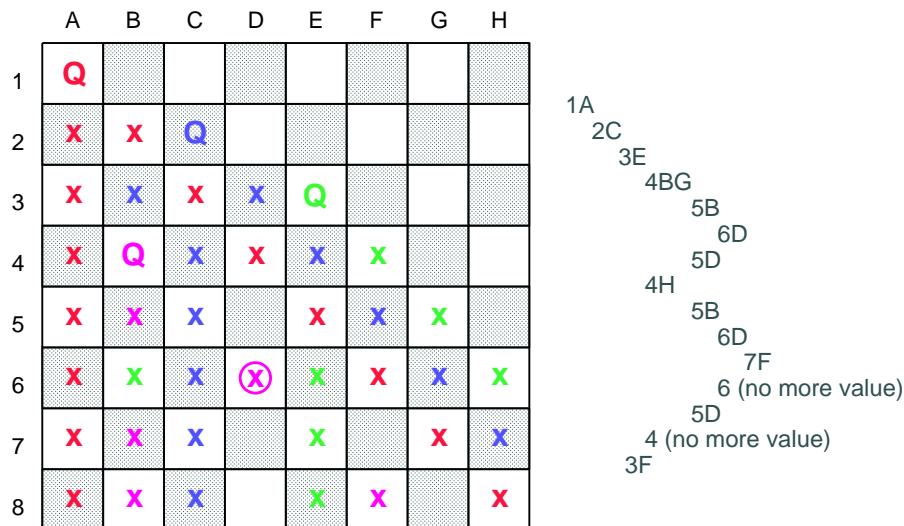
## n-Queens Problem

Place  $n$  queens in an  $n \times n$  chessboard such that no two queens threaten each other.

- Variables  $x_i, i = 1, \dots, n$  with domain  $D_i = \{1, \dots, n\}$  indicating the column of the queen in line  $i$ .
- Constraints
  - $x_i \neq x_j$ , for  $1 \leq i < j \leq n$  (vertical)
  - $x_i \neq x_j + (j - i)$ , for  $1 \leq i < j \leq n$  (diagonal 1)
  - $x_i \neq x_j - (j - i)$ , for  $1 \leq i < j \leq n$  (diagonal 2)

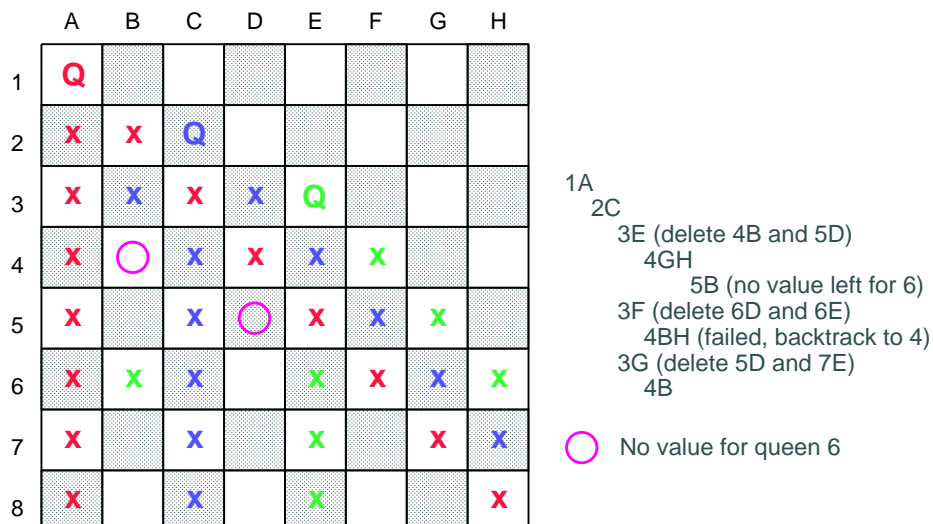
## Forward Checking <sup>(2)</sup>

### Forward Checking



## Partial Lookahead <sup>(3)</sup>

### Partial Lookahead



## Full Lookahead <sup>(4)</sup>

### Full Lookahead

|   | A | B | C | D | E | F | G | H |                      |
|---|---|---|---|---|---|---|---|---|----------------------|
| 1 | Q |   |   |   |   |   |   |   |                      |
| 2 | X | X | Q |   |   |   |   |   |                      |
| 3 | X | X | X | X | Q |   |   |   | 1A<br>2C             |
| 4 | X |   | X | X | X | X |   |   | 3E<br>3F<br>3G<br>3H |
| 5 | X |   | X |   | X | X | X |   | 2D<br>3B<br>3F       |
| 6 | X | X | X |   | X | X | X | X |                      |
| 7 | X |   | X |   | X |   | X | X |                      |
| 8 | X |   | X |   | X |   |   | X |                      |

○ No value for queen 6

### Typical structure of a constraint program

- Declare the variables and their domains
- State the constraints
- Enumeration (labeling)

The constraint solver achieves only local consistency.

In order to get global consistency, the domains have to be enumerated.

### Labeling

- Assigning to the variables their possible values and constructing the corresponding search tree.
- *Important questions*
  1. In which order should the variables be instantiated (variable selection) ?
  2. In which order should the values be assigned to a selected variable (value selection) ?
- Static vs. dynamic orderings
- *Heuristics*

### Dynamic variable/value orderings

- Variable orderings
  - Choose the variable with the smallest domain “*first fail*”
  - Choose the variable with the smallest domain that occurs in most of the constraints “*most constrained*”
  - Choose the variable which has the smallest/largest lower/upper bound on its domain.

- Value orderings
  - Try first the minimal value in the current domain.
  - Try first the maximal value in the current domain.
  - Try first some value in the middle of the current domain.

### Some constraint programming systems

| System     | Avail.          | Constraints   | Language          | Web site   |
|------------|-----------------|---|-------------------|--|
| B-prolog   | comm.           | FinDom  | Prolog            | <a href="http://www.probp.com">www.probp.com</a>   |
| CHIP       | comm.           | FinDom,<br>Boolean,<br>Linear $\mathbb{Q}$            | Prolog,<br>C, C++ | <a href="http://www.cosytec.com">www.cosytec.com</a>   |
| Choco      | free            | FinDom  | Java              | <a href="http://choco.emn.fr">choco.emn.fr</a>   |
| Eclipse    | free non-profit | FinDom,<br>Hybrid                                     | Prolog            | <a href="http://eclipseclp.org">eclipseclp.org</a>   |
| Gecode     | free            | FinDom  | C++               | <a href="http://www.gecode.org">www.gecode.org</a>   |
| GNU Prolog | free            | FinDom  | Prolog            | <a href="http://gnu-prolog.inria.fr">gnu-prolog.inria.fr</a>   |
| ILOG       | comm.           | FinDom,<br>Hybrid                                     | C++,<br>Java      | <a href="http://www-01.ibm.com/software/integration/optimization/cplex-cp-optimizer/">www-01.ibm.com/software/integration/optimization/cplex-cp-optimizer/</a> |
| JaCoP      | free            | FinDom  | Java              | <a href="http://jacop.osolpro.com">jacop.osolpro.com</a>   |
| MiniZinc   | free            | FinDom<br>Arithmetic                                  |                   | <a href="http://g12.cs.mu.oz.au/minizinc">g12.cs.mu.oz.au/minizinc</a>   |
| Mozart     | free            | FinDom  | Oz                | <a href="http://www.mozart-oz.org">www.mozart-oz.org</a>   |
| NCL        | comm.           | FinDom  |                   | <a href="http://www.enginest.com">www.enginest.com</a>   |
| Prolog IV  | free            | FinDom,<br>Arithmetic                                 | Prolog            | <a href="http://prolog-heritage.org">prolog-heritage.org</a>   |
| SCIP       | free            | Hybrid  |                   | <a href="http://scip.zib.de">scip.zib.de</a>   |
| Sicstus    | comm.           | FinDom,<br>Boolean,<br>linear $\mathbb{R}/\mathbb{Q}$ | Prolog            | <a href="http://www.sics.se/sicstus/">www.sics.se/sicstus/</a>   |