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Discrete Mathematics for Bioinformatics (P1)

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Exercises 9 + 10

Exercise 9 (until January 9th)

1. **(PORTA − Polyhedron Representation Transformation Algorithm)**

   Install the PORTA package and read the manpages (http://www2.iwr.uni-heidelberg.de/groups/comopt/software/PORTA).

   Given the following ILP:

$$
\begin{array}{llllllllll}
\max & & x_1 & + & x_2 & + & x_3 & + & x_4 & \\
\text{w.r.t.} & & & & & & & & & \\
 & & x_1 & + & x_2 & + & x_3 & \leq & 2 & \\
 & & x_1 & + & x_2 & + & x_4 & \leq & 2 & \\
 & & & & x_3 & + & x_4 & \leq & 1 & \\
\end{array}
$$

$$x_1, x_2, x_3, x_4 \quad integral$$

   (a) Solve the LP relaxation with a solver (e.g lp-solve or soplex).

   (b) Generate all feasible integral points using program *vint* (PORTA package).

   (c) Transform the point representation into the halfspace representation using program *traf* (PORTA package).

   (d) Solve the resulting linear program again with your lp solver.

2. **Branch and Bound**

$$
\begin{array}{llllllllll}
\max & & 8x_1 & + & 11x_2 & + & 6x_3 & + & 4x_4 & \\
\text{w.r.t.} & & & & & & & & & \\
 & & 5x_1 & + & 7x_2 & + & 4x_3 & + & 3x_4 & \leq & 14 \\
\end{array}
$$

$$x_1, x_2, x_3, x_4 \in \{0, 1\}$$

   (a) Solve the LP relaxation with a solver (e.g lp-solve or soplex).

   (b) Apply branch and bound to find the optimal solution to the ILP.

Exercise 10 (until January 13th)

1. **Critical Mixed Cycles (NIVEAU II)**
   Prove the following lemma (see lecture script):

   A subset $T \subseteq E$ is a trace, if and only if $G' = (V, T, H)$ does not contain a critical mixed cycle.
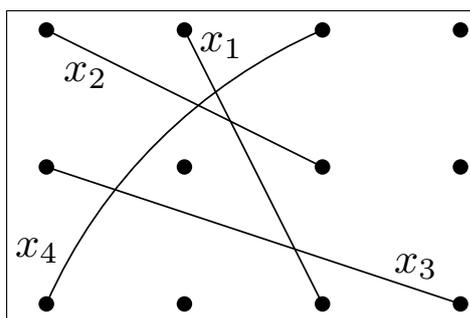
2. **Branch and Cut (NIVEAU I)**
   Apply the cutting plane method to compute an optimal alignment of two sequences "'$ACCA$"' and "'$CACA$"' where a match scores 1 and a mismatch or gap scores 0:

   (a) Draw the alignment graph, the conflict graph, and the pair graph.

   (b) Now start with the trivial (relaxed) LP and add successively clique inequalities which you can find on the longest paths in the pair graph that is labeled with the solution of the last step. Repeat this until you get the optimal alignment.

3. **Branch and Cut (NIVEAU I)**
   Given the following alignment graph:

   All edges have weight 1.

   (a) Try to solve the alignment problem by using branch-and-cut: Add mixed cycle inequalities (see the 'shortest path' method in the script, page 18) to the corresponding (relaxed) LP. Can you reach an optimal solution for the ILP without branching?

   (b) Now use branching to solve the problem.

   (c) Instead of branching, just add the inequality

   $$x_1 + x_2 + x_3 + x_4 \leq 2$$

   Can you solve the ILP now?

   (d) Prove that the inequality in (c) is facet-defining.