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Discrete Mathematics for Bioinformatics (P1)

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Exercises 11

1. **Lagrangean Relaxation I (NIVEAU I)**
   Consider the following problem

$$\begin{array}{rrcrcr} \min & 2x_1 & - & 3x_2 & & \\ \text{w.r.t.} & 3x_1 & - & 4x_2 & \geq & -6 \\ & -x_1 & + & x_2 & \leq & 2 \\ & 6x_1 & + & 2x_2 & \geq & 3 \\ & 6x_1 & + & x_2 & \leq & 15 \\ & & & x_1, x_2 & \geq & 0 \\ & & & x_1, x_2 & \in & \mathbb{Z} \end{array}$$

   (a) Draw the corresponding polytope and determine graphically the optimal solution Z_{IP} of the original problem and Z_{LP}, the solution of the LP-relaxation.

   (b) Now apply lagrangean relaxation by relaxing the first inequality. Draw the polytope of the relaxed ILP. Determine the set X of feasible solutions for the relaxed problem.

   (c) The new objective function is then:

$$Z(P) = \min_{(x_1,x_2)\in X} 2x_1 - 3x_2 + p(-6 - 3x_1 + 4x_2)$$

   .

   Calculate $Z_D = \max_{p\geq 0} Z(p)$ and compare this value to Z_{IP} and Z_{LP}. (To obtain Z_D, draw the graphs of the function $f(p) = 2x_1 - 3x_2 + p(-6 - 3x_1 + 4x_2)$ for all $(x_1, x_2) \in X$.)

   (d) repeat a-c for the objective functions $-x_1 + x_2$ and $-x_1 - x_2$ and compare Z_{LP}, Z_D, and Z_{IP}.

2. **Lagrangean Relaxation II (NIVEAU I)**
   Prove Lemma 1 (see script page 4001) stating that (in case of a minimization problem) if $\lambda \geq 0$, then $Z(\lambda) \leq Z_{IP}$, where Z_{IP} is the optimal value of an original ILP and $Z(\lambda)$ is the optimal value of the relaxed problem for a given value of the Lagrangean multiplier λ.

3. **Structural alignment (NIVEAU I)**

   Prove the two Lemma from the lecture:

   Let $G = (V, E, H, I)$ be a SEAG with n alignment edges and m interaction matches. Then

   - $P_{\mathcal{R}}(G)$ is full-dimensional and
   - the inequality $x_i \leq 1$ is facet-defining iff there is no $e_j \in E$ in conflict with e_i.

   Let $G = (V, E, H, I)$ be a SEAG with n alignment edges and m interaction matches. Then

   - The inequality $x_i \geq 0$ is facet-defining iff e_i is not contained in an interaction match.
   - For each interaction match $m_{i,j}$ the inequality $x_{i,j} \geq 0$ is facet-defining.