Prof. Dr. Alexander Bockmayr,
Prof. Dr. Knut Reinert,
Sandro Andreotti

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Discrete Mathematics for Bioinformatics (P1)

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Exercises 5

1. **Tree decomposition (Niveau I)**

   How large is the largest piece of a any tree decomposition for a graph G of n nodes if G is a clique? Prove your answer.

2. **Tree decomposition (Niveau II)**

   Prove the following theorem:
   If G contains a $(w+1)$-linked set of size at least $3w$, then G has tree-width at least w.

   Hint: Suppose, by way of contradiction, that G has a $(w+1)$-linked set X of size at least $3w$, and it also has a nonredundant TD $(T; \{V_t\})$ of width less than w. The idea of the proof is to find a piece V_t that is "centered" with respect to X, so that when some part of V_t is deleted from G, one small subset of X is separated from another.

3. **Bellman-Ford (Niveau I)**

   (a) Use the Bellman-Ford algorithm to determine the shortest path from source A to any other node in the graph.

   (b) Let $D = (V, A), n = |V|$ be a directed graph. Prove that D contains a circuit of negative length reachable from s if and only if $f_n(v) \neq f_{n-1}(v)$, for some $v \in V$, where $f_k(v) = \min\{l(P) | P \text{ is an } s - v \text{ walk traversing at most } k \text{ arcs}\}$