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Discrete Mathematics for Bioinformatics (P1)

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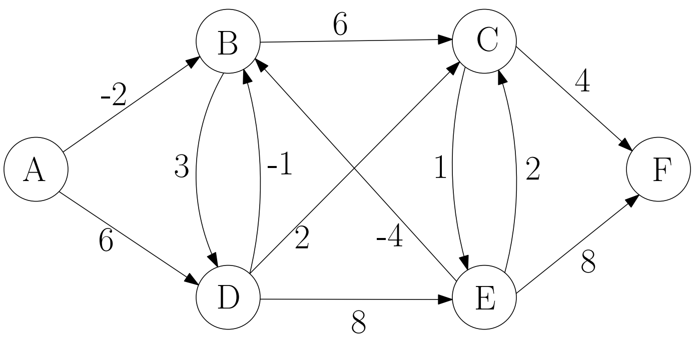
Exercises 5

1. **Bellman-Ford (Niveau I)**

   (a) Use the Bellman-Ford algorithm to determine the shortest path from source A to any other node in the graph.

   (b) Let $D = (V, A), n = |V|$ be a directed graph. Prove that D contains a circuit of negative length reachable from s if and only if $f_n(v) \neq f_{n-1}(v)$, for some $v \in V$, where $f_k(v) = \min\{l(P)|P \text{ is an } s - v \text{ walk traversing at most } k \text{ arcs}\}$



2. **Network Flow (Niveau II)** Prove the Theorem:
   For a network (V, E, s, t) with capacities cap : $E \to \mathbb{R}_+$ the maximum value of a flow is equal to the minimum capacity of an (s, t)-cut:

$$\max\{\text{val}(f) \mid f \text{ is a flow}\} = \min\{\text{cap}(S, T) \mid (S, T) \text{ is an } (s, t)\text{-cut}\}$$

   Hint: Show that the following conditions are equivalent:

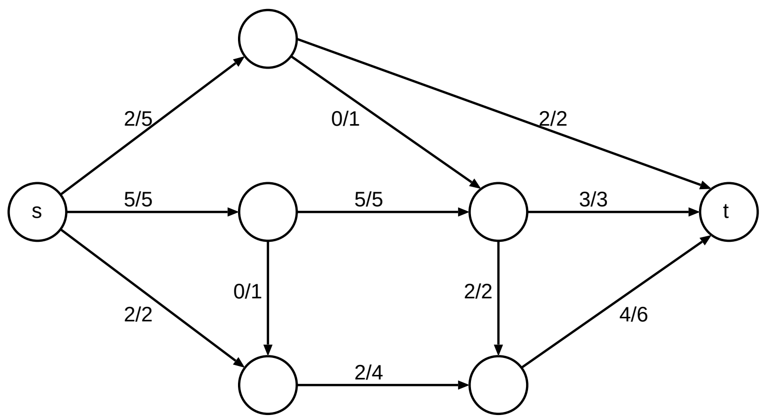
   (a) f is a maximum flow.

   (b) The residual network G_f contains no augmenting path.

   (c) $\text{val}(f) = \text{cap}(S, T)$ for some cut (S, T) of G

3. **Ford-Fulkerson (Niveau I)**

   (a) Use the Ford-Fulkerson algorithm to find a maximum flow in the network



   Start with the initial flow f. An edge label f/c means initial flow f and capacity c.

   (b) Find a minimum cut proving the maximality of the flow.

4. **Matching and Bipartite Graphs (Niveau I)**

   (a) Apply the matching augmenting algorithm for bipartite graphs to the graph below and compute a maximum cardinality matching from the initial matching.