Segment Match Refinement

The function refine(\mathcal{M}) will make a k-partite graph with node sets $V^0 \dots V^{k-1}$ and edges E, which defines boundaries of the minimal resolved refinement of \mathcal{M} .

```
\begin{aligned} \operatorname{def} & \operatorname{cut}(a) \colon \\ & \mathcal{L} = \left\{ M^k = \left( S^i_{u,v}, S^j_{x,y} \right) | u < a < v \lor x < a < y \right\} \\ & \operatorname{for} & M^k \in \mathcal{L} \colon \\ & b = \operatorname{match} & \operatorname{position} & \operatorname{of} & a & \operatorname{in} & \operatorname{sequence} & l & \operatorname{given} & \operatorname{by} & M^k \\ & \operatorname{if} & b \notin V^l \colon \\ & & \operatorname{insert} & b & \operatorname{into} & V^l \\ & & \operatorname{insert} & \operatorname{edge} & (k,a,b) & \operatorname{into} & E \\ & & \operatorname{cut}(b) \end{aligned} \operatorname{def} & \operatorname{refine}(\mathcal{M}) \colon \\ & V^i = \operatorname{boundary} & \operatorname{positions} & \operatorname{of} & \operatorname{segments} & \operatorname{in} & \operatorname{sequence} & i \\ & \operatorname{for} & M^k = \left( S^i_{u,v}, S^j_{x,y} \right) \in \mathcal{M} \colon \\ & & \operatorname{insert} & \operatorname{edge} & \left( \mathbf{k}, \mathbf{u}, \mathbf{x} \right) & \operatorname{and} & \left( \mathbf{k}, \mathbf{v}, \mathbf{y} \right) & \operatorname{into} & E \\ & & \operatorname{for} & \operatorname{boundary} & \operatorname{position} & w & \operatorname{of} & M^k \colon \\ & & & \operatorname{cut}(\mathbf{w}) \\ & \operatorname{lexicographically} & \operatorname{order} & \mathsf{E} \end{aligned}
```

A pair of consecutive edges (k,u,v) and (l,x,y) in E, with k=l defines a new segment match $M=\left(S_{u,x}^i,S_{v,y}^j\right)$ between sequences i and j. If you did not note the sequences the positions are from, you just have to look between which sequences the segment match i was defined.